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Abstract
We study the land and stock markets in Japan circa 1990 and in 2013. While the Nikkei stock average in the late 1980s and its -48% crash in 1990 is generally recognized as a financial market bubble, a bigger bubble and crash was in the land market. The crash in the Nikkei which started on the first trading day of 1990 was predictable in April 1989 using the bond-stock earnings yield model which signaled a crash but not when. We show that it was possible to use the change point detection model based solely on price movements for profitable exits of long positions both circa 1990 and in 2013.

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JEL classification: G01, C11

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Land and Stock Bubbles, Crashes and Exit Strategies in Japan Circa 1990 and in 2013

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We study the land and stock markets in Japan circa 1990 and in 2013. While the Nikkei stock average in the late 1980s and its -48% crash in 1990 is generally recognized as a financial market bubble, a bigger bubble and crash was in the land market. The crash in the Nikkei which started on the first trading day of 1990 was predictable in April 1989 using the bond-stock earnings yield model which signaled a crash but not when. We show that it was possible to use the changepoint detection model based solely on price movements for profitable exits of long positions both circa 1990 and in 2013.

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1. Introduction

The crash of the Japanese stock market in the 1990s and the land markets beginning in 1991 ushered in two decades of deflation, weak economic markets and a lost generation of young people. Various Japanese policies and regulatory constraints exacerbated the poor economic situation and never resolved the basic problem of over leveraging and excessive debt that was a major part of the 1980s buildup.

Our purpose here is not to discuss these issues including the policy not to let the bankrupt companies and individuals go into bankruptcy and start over so that the successful ones would continue and the failures would exit the market or restructure. Rather, our concern is with two issues. First, was it possible to predict the crash and secondly could investors have exited the market unscathed with most of their gains or actually shorted the market successfully using a model based solely on prices? We assume that there was a bubble in both the stock market measured by the Nikkei225 and the land market proxied by the golf course membership indices in various parts of Japan but we do not need there to be an official bubble for our results to hold.

Tests of whether a market is a bubble or not have been studied by Stiglitz (1990) and the papers in that issue of the *Journal of Economic Perspectives* and by Camerer (1989), Scheinkman

This paper is dedicated to the memory of Professor Merton H. Miller who encouraged Ziemba to further study of the golf course membership index markets.
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and Xing (2003), Jarrow et al. (2011), Evanoff et al. (2012), Anderson et al. (2013), Phillips et al. (2013) among others.

Section 2 investigates the question whether or not the Japanese stock market crash was indeed predictable. French and Poterba (1991) and Ziemba and Schwartz (1991) use similar growth models to first justify the high prices of Japanese stocks at that time and the same models to indicated that with different weaker growth parameters a lot lower equilibrium stock market price was predicted. Ziemba and Schwartz (1991) further showed that a long bond minus the reciprocal of past earnings to price ratio model did predict the 1990 crash with a danger signal in April 1989. That model derived from the 1987 US stock market crash predicted correctly 12/12 10%+ declines of the twenty such declines from 1948 to 1988. The other eight declines were for other reasons than high price earnings ratios compared to high long bond interest rates.

This paper discusses the BSEYD (bond stock earnings yield differential) model applied to Japan. A survey of the application of the BSEYD model to the US and other markets since it was discovered in 1988 is in Lleo and Ziemba (2014). The prediction of stock market crises and crashes has been studied by many authors. Major contributions with different models have been made by Buffett (2001), Corcos et al. (2002), Kindleberger and Aliber (2011), Reinhart and Rogoff (2009), Shiller (2000, 2009), Sornette (2003, 2009), Sornette and Zhou (2002), Weigand and Irons (2007), Yan et al. (2012b) and Yan et al. (2012a)

Section 3 describes the changepoint model that we use to determine exits and short entries in bubble type markets. Section 4 applies the model to the Japanese stock market proxied by the Nikkei225 stock average which is price weighted similar to the 30 stock Dow Jones average. Section 5 applies the model to the golf course membership markets in various regions of Japan. Section 6 considers the overall land markets in Japan from 1955 to 1990 before the crash with some discussion regarding this market since 1990. Section 7 considers the 2013 Nikkei market. Section 8 discusses the use of the model for short selling. Section 9 concludes.

2. Was the 1990 Japanese stock market crash predictable?

Starting on the first trading day of 1990, the Nikkei stock average (Nikkei) declined. When it bottomed, the market had fallen 48% from 38,916 at the end of December 1989 to 20,222 on October 1, 1990. This section presents the bond stock earnings yield model that suggested a danger signal in April 1987. This is based on too high interest rates relative to earnings yield, namely the trailing reciprocal of the price-earnings ratio.

In May 1988 I (Ziemba) was invited by Yamaichi Securities to interview to be the first Yamaichi visiting professor of finance at the University of Tsukuba, a Japanese national university. Yamaichi wished to try to establish the study of finance, especially investments, in Japanese universities, which was not generally taught. They established a five-year program with five such visiting professors in succession. My teaching at the university (investments, security market anomalies, futures and options) was supplemented with a two-day a week consulting position in Tokyo some 60 kilometers southwest of Tsukuba at the Yamaichi Research Institute of Yamaichi Securities, then the fourth largest securities firm in Japan and the sixth largest in the world. In my interview I asked if I could study market imperfections (anomalies) and stock market crashes in two study groups with some of the young Yamaichi Research Institute employees who also came up to Tsukuba for my classes.

My proposal was accepted and each study group with about ten eager young students in each group proceeded by me giving lectures on the US experience and they helping me investigate the Japanese situation. We focused on the postwar period 1948 to 1988 and much of what I learned appears in the book *Invest Japan*, Ziemba and Schwartz (1991) and the 1989-1993 research papers of Ziemba, including Schwartz and Ziemba (2000) and Stone and Ziemba (1993). My wife Sandra L. Schwartz and I also wrote the book *Power Japan* (1992) that discussed the Japanese economy. Sandra had a pretty good idea right away that the Japanese policies that let to astronomically high land and stock prices and massive trade surpluses would lead to disaster and they would eventually lose most of the money that they received from selling cars, stereos and the like. We made a list of prestige buildings that the Japanese overpaid for in the 1987-89 era in *Power Japan*. Even at the height of their economic power in 1989 only 3% of Japanese assets were invested abroad.

My study groups started in August 1988 and ended a year later. I was asked to remain as a consultant for the fall of 1988 to complete a factor model discussed in Ziemba and Schwartz (1991) and Schwartz and Ziemba (2000) which was originally presented at a Berkeley Program in Finance meeting in Santa Barbara in September 1992. The factor model used anomaly ideas such as mean reversion, momentum and value embedded in 30 variables to separate and rank stocks by their future mean performance from best to worst for all the 1000+ stocks on the Tokyo Stock Exchange first section which was about 86% of the total capitalization of the Japanese stock market. The model performed well out of sample so was useful for hedge fund long-short trading as well as long only investing. The hedge fund Buchanan Partners in London discovered the model which was discussed in *Invest Japan* when they bought a copy of the book and hired me to consult with them in their warrant trading which was largely long underpriced warrants and short overpriced stocks. Their trading was successful and the model, which was estimated using data during a stock market rise still worked when the decline came since variables such as earnings were the key drivers of the returns. An update of Japanese anomalies to 1994 appears in Comolli and Ziemba (2000). Ziemba's Japanese, US and other anomaly papers are reprinted in Ziemba (2012).

In the crash study group I came up with a simple model in 1988 with only a single variable that being the difference between stock and bond rates of return. The idea was that stocks and bonds compete for investment dollars and, when interest rates are low, stocks are favored and when interest rates are high, bonds are favored. The main thing that I wished to focus on is that when the measure, the difference between these two rates, the long bond yield minus the earnings yield (the reciprocal of the price earnings ratio), was very large, then there was a high chance of a stock market crash, namely a 10%+ fall in the index within one year. The model explains the October 1987 crash. That application is how this idea came to me. Table 1 and Figure 1 show this. The boxes indicate that there is extreme danger in the stock market because 30-year government bond yields are very much higher than usual stock market yields measured by the reciprocal of the last year's reported price earnings ratio. These high interest rates invariably lead to a stock market crash. Here the danger indicator moved across a statistical 95% confidence line in April 1987.[1] The market ignored this signal but did eventually crash in October 1987. While Table 1 uses S&P500 data and Berge et al. (2008) use the US MSCI, both reach the same conclusion. There was a similar signal ignored by most investors in the US S&P500 in 1999 and then a crash that began in August 2000 and a weak stock market in 2001/02 which is discussed in Ziemba (2003).[2]

In 1988-89, I asked one of my young colleagues in my crash study group, Sugheri Iishi, to check this for Japan. There were twenty 10% plus crashes during the forty years, 1948-88, see Table 2. We found that whenever this measure was in the danger zone (that is outside a 95% confidence band), there was a crash of 10% or more from the current level within one year. Not all crashes had the measure in the danger zone but whenever it was there was a crash with no misses. That model was 12/12.

So the measure was successful at predicting future crashes – but when and how deep there was no precise way to know. However, long-run mean reversion (Poterba and Summers (1985))

[1]For a study of this measure for staying in or exiting stock markets from 1975 and 1980 to 2005 in five major countries (US, Japan, UK, Germany and Canada), see Berge et al. (2008). In each country final wealth under the strategy in cash in the danger periods (about 20% of the time) is about double the buy and hold strategy with lower standard deviation risk.

[2]Later Yardeni (1997) reported on a related Fed model which is a special case to the difference model used here. See Lleo and Ziemba (2013) where critiques of the difference model are also presented and discussed. Our aim here is the answer the question: does the model actually predict stock market crashes?

Table 1. S&P500 index, PE ratios, government bond yields and the yield premium over stocks, January 1984 to August 1988. Source: Ziemba and Schwartz (1991)

		S&P Index	PER	(a) 30 Yr G bd	(b) 1/pe,%	(a)-(b)
1986	Jan	208.19	14.63	9.32	6.84	2.48
	Feb	219.37	15.67	8.28	6.38	1.90
	Mar	232.33	16.50	7.59	6.06	1.53
	Apr	237.98	16.27	7.58	6.15	1.43
	May	238.46	17.03	7.76	5.87	1.89
	Jun	245.30	17.32	7.27	5.77	1.50
	Jul	240.18	16.31	7.42	6.13	1.29
	Aug	245.00	17.47	7.26	5.72	1.54
	Sep	238.27	15.98	7.64	6.26	1.38
	Oct	237.36	16.85	7.61	5.93	1.68
	Nov	245.09	16.99	7.40	5.89	1.51
	Dec	248.60	16.72	7.33	5.98	1.35
1987	Jan	264.51	15.42	7.47	6.49	0.98
	Feb	280.93	15.98	7.46	6.26	1.20
	Mar	292.47	16.41	7.65	6.09	1.56
	Apr	289.32	16.22	9.56	6.17	3.39
	May	289.12	16.32	8.63	6.13	2.50
	Jun	301.38	17.10	8.40	5.85	2.55
	Jul	310.09	17.92	8.89	5.58	3.31
	Aug	329.36	18.55	9.17	5.39	3.78
	Sep	318.66	18.10	9.66	5.52	4.14
	Oct	280.16	14.16	9.03	7.06	1.97
	Nov	245.01	13.78	8.90	7.26	1.64
	Dec	240.96	13.55	9.10	7.38	1.72
1988	Jan	250.48	12.81	8.40	7.81	0.59
	Feb	258.10	13.02	8.33	7.68	0.65
	Mar	265.74	13.42	8.74	7.45	1.29
	Apr	262.61	13.24	9.10	7.55	1.55
	May	256.20	12.92	9.24	7.74	1.50
	Jun	270.68	13.65	8.85	7.33	1.52
	Jul	269.44	13.59	9.18	7.36	1.82
	Aug	263.73	13.30	9.30	7.52	1.78

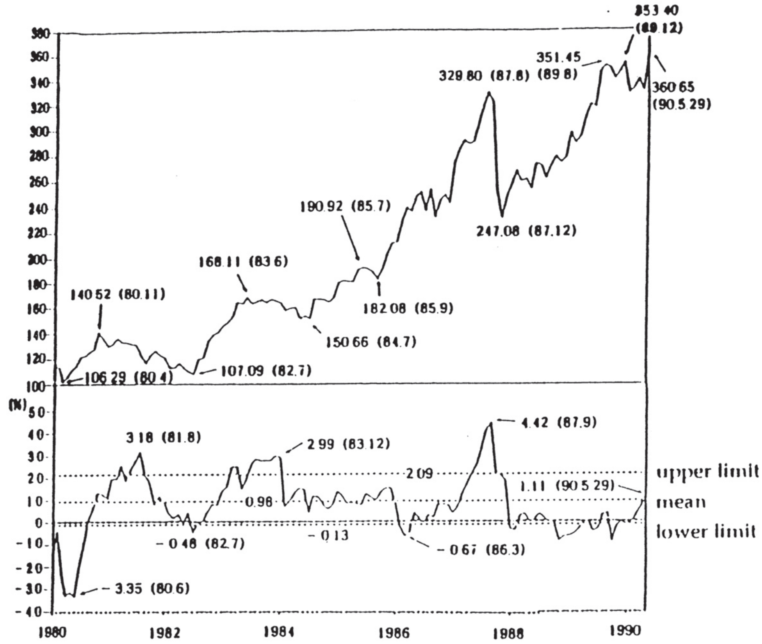
suggests that the longer the bull run is and the more over-priced the measure is, the longer and deeper the decline will probably be. Then one can use the measure as part of an econometric system to estimate future scenarios. The rest of this paper presents a model to exit or enter such markets.

Each time the spread exceeded the 4.23 cutoff (which was higher than 95% confidence) there was a crash. The measure was way in the danger zone in late 1989 and the decline (the 21st crash) began on the first trading day of 1990 with the Nikkei stock average peaking at 38,916. See Figure 2. It is too bad Yamaichi's top management did not listen to Iishi when I sent him up to explain our results in Japanese; there was much greater danger in the market than they thought in 1989. By 1999 Yamaichi Securities was bankrupt and ceased to exist. Figure 3(a) shows why this happened because too high interest rates in 1990 that completely crushed the economy.

The model also indicated that the valuation was still high as of May 29, 1990 at 4.88. Not much later, the 22nd crash began. Interestingly, at the bottom of the 22nd crash on October 1, 1990, the Nikkei was at 20,222, which was almost exactly the mean. Meanwhile, the same calculation on May 29, 1990, for the S&P500 is shown in Figure 1. Indeed, it was cheap, that is below the mean, since the September 1987 peak of 4.42. The May 29, 1990 value of 1.11 was, however, slightly above the mean level and the highest since the late fall of 1987.

Japan has had weak stock and land markets for twenty-three years, since the beginning of 1990.

Figure 1. Bond and stock yield differential model for the S&P500, 1980-1990, Source: Ziemba and Schwartz, 1991



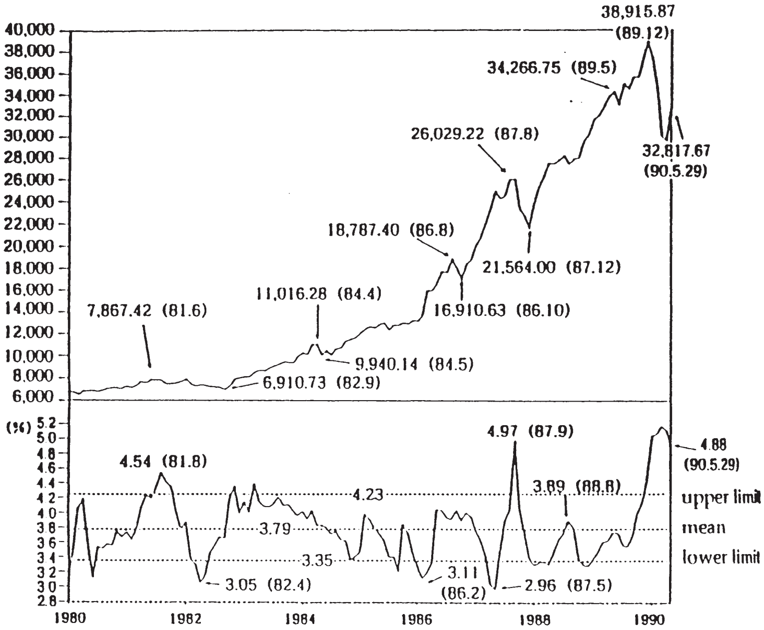
Value of S&P500 for various spread values

Date/level	Spread, %	S&P500
May 29, 1990	1.11	360.65
Mean	0.98	355.00
Upper limit	2.09	415.00
Lower limit	-0.13	309.00

Table 2. The Twenty Corrections of 10% or more on the NSA from 1949 to 1988. Source: Yamaichi Research Institute

	Index Value			Date		
	Peak	Valley	% decrease	Peak	Valley	# Months
1	176.89	85.25	-51.8	01-Sep-49	06-Jul-50	11
2	474.43	295.18	-37.8	4-Feb-53	1-Apr-53	2
3	366.69	321.79	-12.2	6-May-53	3-Jun-53	1
4	595.46	471.53	-20.8	4-May-57	27-Dec-57	8
5	1,829.74	1,258.00	-31.2	18-Jul-61	19-Dec-61	5
6	1,589.76	1,216.04	-23.5	14-Feb-62	29-Oct-62	9
7	1,634.37	1201.26	-26.5	5-Apr-63	18-Dec-63	9
8	1,369.00	1,020.49	-25.5	3-Jul-64	12-Jul-65	13
9	1,588.73	1,364.34	-14.1	1-Apr-66	15-Dec-66	8
10	1,506.27	1,250.40	-17.0	1-Mar-67	11-Dec-67	9 .
11	2,534.45	1,929.64	-23.9	6-Apr-70	27-May-70	2
12	2,740.98	2,227.25	-18.7	13-Aug-71	20-Oct-71	3
13	5,359.74	3,355.13	-37.4	24-Jan-73	9-Oct-74	21
14	4,564.52	3,814.02	-16.4	12-May-75	29-Sep-75	5
15	5,287.65	4,597.26	-13.1	5-Sep-77	24-Nov-77	3
16	8,019.14	6,849.78	-14.6	17-Aug-81	1-Oct-82	14
17	11,190.17	9,703.35	-13.3	4-May-84	23-Jul-84	3
18	18,936.24	15,819.58	-16.5	20-Aug-86	22-Oct-86	2
19	25,929.42	22,702.74	-12.4	17-Jun-87	22-Jul-87	1
20	26,646.43	21,036.76	-21.1	14-Oct-87	11-Nov-87	1
Average			-0.224			6.5

Figure 2. Bond and stock yield differential model for Nikkei, 1980-1990, Source: Ziemba and Schwartz, 1991



Value of Nikkei for various spread values

Date/level	Spread, %	Nikkei
May 29, 1990	4.88	32,818
Mean	3.79	20,022
Upper limit	4.23	23,754
Lower limit	3.35	17,303

There are many factors for this that are political as well as economic. But the rising interest rates for eight full months until August 1990 shown in Figure 3 is one of them. This extreme tightening of an over-levered economy was too much. Cheap and easily available money, which caused the big run-up in asset prices in the 1980s turned into expensive and unavailable money in the 1990s.

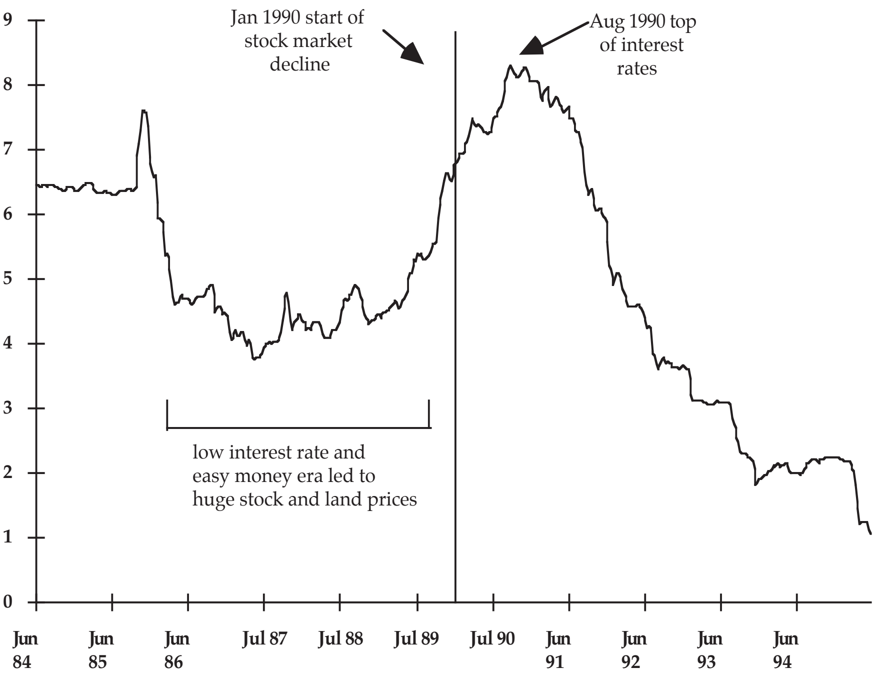
3. The changepoint detection model for exit bubble type markets

The finance and economics literature has research on identifying and timing bubbles. Trading bubble type markets is difficult. It is known that famed bubble trader George Soros shorted the Japanese market too soon in 1988 and lost about $1 billion. As shown in section 2, Ziemba's BSEYD model suggested an exit danger signal in April 1989 with the decline in 1990 of some -48% starting on the first trading day of 1990.

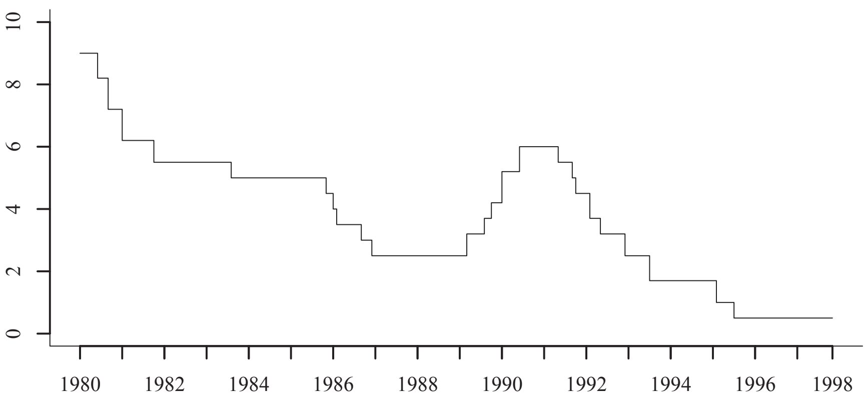
There are however some studies such as those referenced in Section 1 which mostly focus on determining if a particular market is a bubble or not. What we mean by a bubble is a price that is going up just because it is going up! That is one where the price exceeds the fair value.

In this paper we present a Bayesian changepoint detection model that seems to work well timing when to exit a long position. We apply the model to Japanese stock and land markets – the latter proxied by the golf course membership market which has weekly transaction data. The Nikkei225 data is daily. We do not attempt to use the model for various types of land since there is no frequent data available.

The basic idea is that there is a fast rate of growth in prices, then a peak and then a fast decline. The model tries to exit near the peak in prices. It is based on the mathematical theory of *changepoint detection* (or *disorder detection*) for stochastic processes. A changepoint of a



(a) Short term interest rates in Japan, June 1984 to June 1995



(b) Bank of Japan target interest rates 1980 to 1998

Figure 3. Interest rates in Japan

stochastic process is an unknown moment of time when its probabilistic structure changes (e.g. the mean value changes from a positive number to a negative one). The theory studies statistics that allow to detect such changes after they happen.

Changepoint detection methods have been successfully applied in production quality control, radiolocation, information security, and have shown their usefulness. Their history goes back to the pioneering work of Shewhart (1925), and the first fundamental results by Page, Roberts, Shiryaev and others of 1950-60s. Surveys of the history and the recent developments in this field can be found in e. g. the introduction to the book by Poor and Hadjiljadis (2009).

In the financial context, a changepoint may represent a moment when the market starts to decline. It can be identified with a moment of time when the trend of the sequence of the market's index value becomes negative. The objective of the model is to detect this change after it occurs and to close a long position maximizing the gain measured by a given utility function.

We emphasize that the changepoint model does not *predict* a decline of price, but considers the problem of how to detect it *after* it has started, taking into account that temporal declines may be caused by the volatility of the price rather than a real change of the trend. In particular, such a decline need not necessarily be caused by a crash of a bubble, but can be due to e.g. a structural change in an economy or bad news for a company. Thus we do not need the price process to be a bubble in terms of one definition or another.

3.1 *The description of the model*

The model describes the evolution of the value S of an index at moments of time $t = 0, 1, \ldots, T$ driven by a geometric Gaussian random walk with logarithmic mean and variance (μ_1, σ_1^2) up to an unknown moment of time θ and (μ_2, σ_2^2) after θ. The moment θ will be interpreted as the point when the trend of the index changes from an increasing to a decreasing, and is called a *changepoint*. In this paper, we provide only a brief exposition, details can be found in Zhitlukhin (2013); a continuous-time analogue was considered by Shiryaev and Zhitlukhin (2012).

Let $\xi = (\xi_t)_{t=0}^T$ be a sequence of i.i.d.[1] standard normal random variables defined on a probability space $(\Omega, \mathcal{F}, \mathsf{P})$. The sequence $S = (S_t)_{t=0}^T$ is defined by its logarithmic increments via

$$S_0 = 1, \qquad \log \frac{S_t}{S_{t-1}} = \begin{cases} \mu_1 + \sigma_1 \xi_t, & t < \theta, \\ \mu_2 + \sigma_2 \xi_t, & t \geq \theta, \end{cases}$$

where $\mu_1, \mu_2 \in R$, $\sigma_1, \sigma_2 > 0$ are known parameters. The choice of $S_0 = 1$ means that all the prices are expressed relatively to the price at time $t = 0$, which does not reduce the generality of the model.

It is assumed that θ is a random variable defined on $(\Omega, \mathcal{F}, \mathsf{P})$, independent of the sequence ξ_t and taking values in the set $\{1, 2, \ldots, T+1\}$ with known prior probabilities $p_t = \mathsf{P}(\theta = t)$. The value p_1 is the probability that the changepoint appears from the beginning of the sequence S_t , and p_{T+1} is the probability that the changepoint does not appear within the time horizon $[0, T]$. The prior distribution function of θ is denoted by $G(t) = p_1 + p_2 + \ldots + p_t$.

Let $U_\alpha \colon R_+ \to R$, $\alpha \leq 0$, be the family of negative power and logarithmic utility functions:[2]

$$U_\alpha(x) = -x^\alpha \text{ for } \alpha < 0, \qquad U_0(x) = \log x.$$

The problem consists in finding the moment of time τ which maximizes the utility from closing a long position provided one opens it at time $t = 0$ and needs to close before $t = T$.

Let $\mathbb{F} = (\mathcal{F}_t)_{t=0}^T$, $\mathcal{F}_t = \sigma(S_u;\ u \leq t)$ be the filtration generated by the process S. By definition, a moment τ when one closes the position should be a *stopping time* of the filtration $\mathbb{F}$, i. e. τ should be a random variable taking values in the set $\{0, 1, \ldots, T\}$ such that $\{\omega : \tau(\omega) \leq t\} \in \mathcal{F}_t$ for any $t = 0, \ldots, T$. The class of all stopping times $\tau \leq T$ of $\mathbb{F}$ is denoted by $\mathfrak{M}$. The notion of a stopping time reflects the concept that a decision to close the position at time t should be based only on the information obtained from the observed values $S_0, S_1, \ldots, S_t$ and should not rely on the future values S_{t+1}, S_{t+2}, which are unknown at time t.

The problems of optimal closing of a long positions with respect to the utility function U_α is formulated as the *optimal stopping problem*

$$V_\alpha = \sup_{\tau \in \mathfrak{M}} \mathsf{E} U_\alpha(S_\tau). \tag{1}$$

[1]Independent and identically distributed.
[2]These functions are from capital growth theory and are known to maximize long run asymptotic growth of wealth; see e.g., Kelly (1956), Breiman (1961), and MacLean et al. (2010). Log is full Kelly and negative power are fractional Kelly strategies blending cash with the max Elog portfolio to lower risk and the long run growth rate.

Its solution consists in finding the *optimal stopping time* τ_α, at which the supremum is attained (it is shown that such a stopping time exists).

It is assumed that $\mu_1 > -\alpha\sigma_1^2/2$ and $\mu_2 < -\alpha\sigma_2^2/2$. Under these assumptions the sequence $\{U_\alpha(S_t)\}_t$ increases on average if the logarithmic increments of S are i.i.d. $\mathcal{N}(\mu_1, \sigma_1^2)$ random variables, and decreases on average if they are i.i.d. $\mathcal{N}(\mu_2, \sigma_2^2)$ random variables. Consequently, the random variable θ represents the moment of time when holding the index value starts to decrease.

In order to formulate our main result, Theorem 1, we introduce auxiliary notation. Let $\psi = (\psi_t)_{t=0}^T$ denote the Shiryaev–Roberts statistic of the sequence of values S_t:

$$\psi_0 = 0, \qquad \psi_t = \frac{\sigma^1}{\sigma^2}(p_t + \psi_{t-1})\, e^{\frac{(X_t-\mu_1)^2}{2\sigma_2^2} - \frac{(X_t-\mu_2)^2}{2\sigma_2^2}}, \quad t = 1, \dots, T.$$

where $X = (X_t)_{t=1}^T$ denote the logarithmic increments of the index values, $X_t = \log(S_t/S_{t-1})$.

Define recurrently the family of functions $V_\alpha(t,x)$ for $\alpha \le 0$, $t = T, T-1, \dots, 0$, $x \ge 0$ as follows. For $\alpha = 0$ let

$$V_0(T,x) \equiv 0, \quad V_0(t,x) = \max\bigl\{0,\ \mu_2(x + p_{t+1}) + \mu_1(1 - G(t+1)) + f_0(t,x)\bigr\},$$

where the function $f_0(t,x)$ is given by

$$f_0(t,x) = \frac{1}{\sigma_1\sqrt{2\pi}} \int_{\mathbb{R}} V_0\Bigl(t+1,\ \frac{\sigma_1}{\sigma_2}(p_{t+1} + x)\, e^{\frac{(z-\mu_1)^2}{2\sigma_1^2} - \frac{(z-\mu_2)^2}{2\sigma_2^2}}\Bigr) e^{-\frac{(z-\mu_1)^2}{2\sigma_1^2}}\, dz.$$

For $\alpha < 0$ define

$$V_\alpha(T,x) \equiv 0, \quad V_\alpha(t,x) = \max\bigl\{0,\ \beta^t\bigl[(\gamma-1)(x + p_{t+1}) + (\beta-1)(1 - G(t+1))\bigr] + f_\alpha(t,x)\bigr\},$$

where $\beta = e^{\alpha\mu_1 + \alpha^2\sigma_1^2/2}$, $\gamma = e^{\alpha\mu_2 + \alpha^2\sigma_2^2/2}$ and

$$f_\alpha(t,x) = \frac{1}{\sigma_1\sqrt{2\pi}} \int_{\mathbb{R}} V_\alpha\Bigl(t+1,\ (p_{t+1} + x)\cdot \frac{\sigma_1}{\sigma_2} e^{\frac{(z-\mu_1)^2}{2\sigma_1^2} - \frac{(z-\mu_2)^2}{2\sigma_2^2}}\Bigr) e^{-\frac{(z-\mu_1-\alpha\sigma_1^2)^2}{2\sigma_1^2}}\, dz.$$

Theorem 3.1 (Zhitlukhin (2013), Chapter 3)**:** *The following stopping time is optimal in problem* (1)*:*

$$\tau_\alpha^* = \inf\{0 \le t \le T : \psi_t \ge b_\alpha^*(t)\}, \tag{2}$$

where the stopping boundary $b_\alpha^*(t)$*,* $t = 0, \dots, T$*, is*

$$b_\alpha^*(t) = \inf\{x \ge 0 : V_\alpha(t,x) = 0\}.$$

Theorem 3.1 states that the optimal stopping time is the first moment of time when the Shiryaev–Roberts statistic exceeds the time-dependent threshold $b_\alpha^*(t)$. To find the function $b_\alpha^*(t)$ numerically, one first computes recurrently the functions $V_\alpha(t,x)$ for $t = T, T-1, \dots, 0$ and then finds their minimal roots $x \ge 0$. The algorithm is described in Zhitlukhin (2013) along with the estimation of its computational error.

It is also possible to express the optimal stopping time through the *posterior probability process* $\pi = (\pi_t)_{t=0}^T$ defined as the conditional probability $\pi_t = \mathsf{P}(\theta \le t \mid \mathcal{F}_t)$. From Bayes formula (see details in Zhitlukhin (2013)),

$$\psi_t = \frac{\pi_t}{1-\pi_t}(1 - G(t)), \qquad \pi_t = \frac{\psi_t}{\psi_t + 1 - G(t)}.$$

Consequently, the optimal stopping time in problem (1) can be represented in the form

$$\tau_\alpha^* = \inf\{0 \le t \le T : \pi_t \ge \tilde{b}_\alpha^*(t)\}, \text{ where } \tilde{b}_\alpha^*(t) = \frac{b_\alpha^*(t)}{b_\alpha^*(t) + 1 - G(t)}.$$

This representation provides a clear interpretation of the optimal stopping time: one should close a long position as soon as one becomes sufficiently confident that the disorder has already happened, which quantitatively means the posterior probability π_t exceeds the threshold $\tilde{b}_\alpha^*(t)$.

3.2 *Application of the model to market data*

We observe an index such that initially its value has a positive trend. The time interval between two consecutive index values can be arbitrary. It is assumed that a long position is opened at time $t = 0$ and should be closed by a fixed time $t = T$. The aim is to find the right moment when to close the position.

The observable values of the index are represented by the sequence $S_0, S_1, \ldots, S_T$, which is assumed to follow the above model of a geometric Gaussian random walk with a changepoint. In the applications below, we assume θ is uniformly distributed in the set $\{1, 2, \ldots, T\}$, which in some sense corresponds to the "worst case" (since the uniform distribution has the maximum entropy among all distributions on a finite set).

The parameters μ_1, σ_1 of the model are estimated using from the past data $S_0, S_{-1}, S_{-2}, \ldots$, S_{-t_0}, where t_0 is some fixed constant, assuming that the logarithmic increments of the sequence $\{S_t\}_{t=-t_0}^0$ are i.i.d $\mathcal{N}(\mu_1, \sigma_1)$ random variables.

The choice of the values μ_2 and σ_2 is subjective, as we do not know the "future" values of the drift and volatility after they change. In the applications below, we let $\sigma_2 = \sigma_1$ and consider several choices of the drift parameter[1]: $\mu_2 = -0.5\mu_1$, $\mu_2 = -\mu_1$, $\mu_2 = -2\mu_1$, $\mu_2 = -3\mu_1$, and $\mu_2 = -5\mu_1$, which however do not give any significantly different results.

We also vary the utility function U_α and the final moment of time T, and apply the model to $U_0(x) = \log x$, $U_{-1/2}(x) = -1/\sqrt{x}$ and $U_{-1}(x) = -1/x$ (i.e. the full Kelly rule, the 2/3 Kelly rule and the half Kelly rule assuming log normally distributed asset) and T corresponding to the end of 1992, 1990 and 1989 for the 1990 crash and T corresponding to the end of 2013 for the recent Nikkei225 data. When T corresponds to the end of 1989 no crash occurs on the observable time horizon, so the model should close the position close to the global maximum.

4. The Japanese stock market bubble

4.1 *Background*

The Japanese stock market was closed after World War II ended in 1945 until its reopening in 1948. From 1948 to 1988 there was a huge rise in the stock market measured by the Nikkei225 price weighted stock index as well as the Topix value weighted index of more than 1000 stocks. A steady increase in quality and quantity of equipment and automobiles of various kinds led to an enormous inflow of financial assets. These in turn were invested primarily in Japanese stocks and land. The land and stock markets were greatly intertwined as discussed by Ziemba (1991) and Stone and Ziemba (1993); see also section 6. To get an idea of the price pressure on Japanese land prices, consider that in the late 1980s:

- some 120 million people lived in Japan in an area the size of Montana,

[1]It is well known that getting the means right is much more important than the variances, see Chopra and Ziemba (1993). Thus we focus on the analysis of the performance of the model for various drift values, but we assume that the variance stays constant.

- only 5% of the land was used to house the people, buildings and factories because most of the land is mountainous,
- most of the land was owned by large corporations but 60% of Japanese families and 55% of those in Tokyo owned their own home,
- there was massive savings by households,
- only some 3% of Japanese assets were invested abroad despite great fear in the west and some very public purchases at inflated prices of expensive property such as the Pebble Beach golf course,
- low interest rates in the mid to late 1980s fueled the stock and land booms,
- the Nikkei225 rose 220 times in yen and 550 times in US dollars from 1948 to 1988 yet there were twenty 10%+ declines in the Nikkei225, see Table 2.

Figure 4 shows the Nikkei stock average from 1984 to 2014 and Figure 3(b) shows the Bank of Japan target interest rate from 1980 to 1998.

Figure 4. Nikkei-225 from 1984 to 2013

The Nikkei peaked at the end of December 1989 at 39,816. In section 2, the bond-stock earnings yield model was shown to go into the danger zone in April 1989. That model suggested that a large decline or crash was coming but not when. To apply the changepoint detection model a maximum final time horizon needs to be specified. We apply it for three time horizons, namely to the ends of 1989, 1990 and 1992. In section 6 we apply the model to the Nikkei225 in 2013 .

4.2 *Nikkei stock average in the 1980-90s*

The model was applied to the Nikkei stock average with six different starting dates (January 1987, July 1987, July 1988, January 1989, July 1989, October 1989), four different values of the ratio μ_2/μ_1 (-0.5, -1.0, -2.0, -5.0), three different utility functions $U_0(x) = \log x$, $U_{-1/2}(x) = 1/\sqrt{x}$, $U_{-1}(x) = -1/x$, and three different time horizons (the ends of 1989, 1990, and 1992). In all the cases, $\sigma_2 = \sigma_1$.

The prices are daily closing prices; the parameters μ_1 and σ_1 are estimated using the 100 previous index values before the entering date. The data appears in Tables 3-5 and Figure 5. The tables show the exit dates obtained by applying the model for the corresponding values of the parameters, and the ratio of the price at these dates to the maximal closing price (on December 29, 1989). The figures present the four entry dates in 1988-1989 marked by blue circles and the corresponding exit dates marked by red squares for the value of the parameters $\alpha = 0$, $\mu_2 = -\mu_1$. The graphs are normalized in a such way that the price on the first trading day of 1984 is 100.

The results show for $\alpha = 0$ (log utility) exits from 54.5% of the maximum for entries in 1987 to 94.7% for entries in 1988 and 85.6% for entries in 1989 of the global maximum of the market.

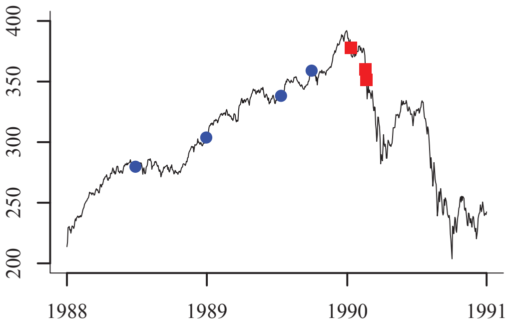
When T=1989, the exit percent of peak value depends mostly on the entry time. Later entries exit closer to the peak. The T=1992 results vary greatly depending on the entry date with the early entry dates exiting well below the peak and later entry dates exiting closer to the global maximum. The choice of α and the value of the ratio μ_1/μ_2 does not change the results as we are just searching for the exit point, not trading after that, see Tables 3 to 5. In general the change point model yields a profit based on the entry and exit points; see Figure 5(a) and 5(b). But it is possible that late entries close to the peak as in Figure 5(c) with T=1992 result in a trading loss because of the sharp decline after the peak.

Table 3.: The exit dates for Nikkei stock average, T = end of 1990

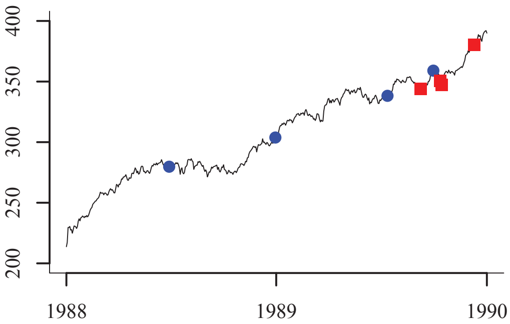
Enter	μ_1	σ	$\lvert\mu_2/\mu_1\rvert$			
			0.5	1.0	2.0	5.0
			$\alpha = 0.0$			
26.12.86	0.00086	0.01300	26.02.90 (85.6%)	26.02.90 (85.6%)	23.02.90 (89.7%)	20.10.87 (56.3%)
30.06.87	0.00191	0.01114	23.02.90 (89.7%)	11.11.87 (54.1%)	20.10.87 (56.3%)	20.10.87 (56.3%)
30.06.88	0.00163	0.00585	16.01.90 (94.7%)	16.01.90 (94.7%)	16.01.90 (94.7%)	16.01.90 (94.7%)
28.12.88	0.00064	0.00634	26.02.90 (85.6%)	23.02.90 (89.7%)	21.02.90 (91.8%)	18.01.90 (94.4%)
30.06.89	0.00039	0.00602	26.02.90 (85.6%)	23.02.90 (89.7%)	21.02.90 (91.8%)	16.01.90 (94.7%)
29.09.89	0.00045	0.00497	26.02.90 (85.6%)	21.02.90 (91.8%)	21.02.90 (91.8%)	16.01.90 (94.7%)
			$\alpha = -0.5$			
26.12.86	0.00086	0.01300	26.02.90 (85.6%)	23.02.90 (89.7%)	23.02.90 (89.7%)	20.10.87 (56.3%)
30.06.87	0.00191	0.01114	26.02.90 (85.6%)	10.11.87 (55.7%)	20.10.87 (56.3%)	20.10.87 (56.3%)
30.06.88	0.00163	0.00585	16.01.90 (94.7%)	16.01.90 (94.7%)	16.01.90 (94.7%)	16.01.90 (94.7%)
28.12.88	0.00064	0.00634	26.02.90 (85.6%)	23.02.90 (89.7%)	21.02.90 (91.8%)	16.01.90 (94.7%)
30.06.89	0.00039	0.00602	26.02.90 (85.6%)	23.02.90 (89.7%)	21.02.90 (91.8%)	16.01.90 (94.7%)
29.09.89	0.00045	0.00497	26.02.90 (85.6%)	21.02.90 (91.8%)	21.02.90 (91.8%)	16.01.90 (94.7%)
			$\alpha = -1.0$			
26.12.86	0.00086	0.01300	23.02.90 (89.7%)	23.02.90 (89.7%)	11.11.87 (54.1%)	20.10.87 (56.3%)
30.06.87	0.00191	0.01114	04.01.88 (54.5%)	10.11.87 (55.7%)	20.10.87 (56.3%)	20.10.87 (56.3%)
30.06.88	0.00163	0.00585	18.01.90 (94.4%)	18.01.90 (94.4%)	16.01.90 (94.7%)	16.01.90 (94.7%)
28.12.88	0.00064	0.00634	26.02.90 (85.6%)	23.02.90 (89.7%)	21.02.90 (91.8%)	16.01.90 (94.7%)
30.06.89	0.00039	0.00602	26.02.90 (85.6%)	23.02.90 (89.7%)	21.02.90 (91.8%)	16.01.90 (94.7%)
29.09.89	0.00045	0.00497	26.02.90 (85.6%)	21.02.90 (91.8%)	21.02.90 (91.8%)	16.01.90 (94.7%)

Table 4.: The exit dates for Nikkei stock average, T = end of 1989

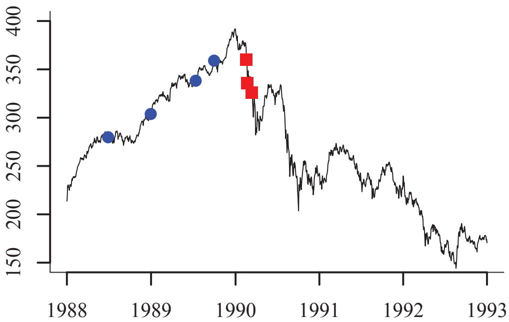
Enter	μ_1	σ	$\lvert\mu_2/\mu_1\rvert$			
			0.5	1.0	2.0	5.0
			$\alpha = 0.0$			
26.12.86	0.00086	0.01300	23.08.89 (89.7%)	17.07.89 (86.0%)	11.11.87 (54.1%)	20.10.87 (56.3%)
30.06.87	0.00191	0.01114	11.11.87 (54.1%)	20.10.87 (56.3%)	20.10.87 (56.3%)	20.10.87 (56.3%)
30.06.88	0.00163	0.00585	29.06.89 (84.7%)	16.10.89 (88.6%)	16.10.89 (88.6%)	16.10.89 (88.6%)
28.12.88	0.00064	0.00634	16.10.89 (88.6%)	07.09.89 (87.8%)	07.09.89 (87.8%)	12.10.89 (89.4%)
30.06.89	0.00039	0.00602	07.11.89 (90.6%)	12.10.89 (89.4%)	06.09.89 (88.1%)	30.08.89 (88.6%)
29.09.89	0.00045	0.00497	19.12.89 (98.8%)	11.12.89 (97.0%)	16.10.89 (88.6%)	12.10.89 (89.4%)
			$\alpha = -0.5$			
26.12.86	0.00086	0.01300	17.07.89 (86.0%)	29.06.89 (84.7%)	10.11.87 (55.7%)	20.10.87 (56.3%)
30.06.87	0.00191	0.01114	11.11.87 (54.1%)	20.10.87 (56.3%)	20.10.87 (56.3%)	20.10.87 (56.3%)
30.06.88	0.00163	0.00585	06.09.89 (88.1%)	16.10.89 (88.6%)	16.10.89 (88.6%)	16.10.89 (88.6%)
28.12.88	0.00064	0.00634	12.10.89 (89.4%)	07.09.89 (87.8%)	06.09.89 (88.1%)	08.09.89 (87.7%)
30.06.89	0.00039	0.00602	07.11.89 (90.6%)	12.10.89 (89.4%)	06.09.89 (88.1%)	30.08.89 (88.6%)
29.09.89	0.00045	0.00497	19.12.89 (98.8%)	11.12.89 (97.0%)	16.10.89 (88.6%)	12.10.89 (89.4%)
			$\alpha = -1.0$			
26.12.86	0.00086	0.01300	28.06.89 (85.4%)	15.06.89 (84.6%)	20.10.87 (56.3%)	20.10.87 (56.3%)
30.06.87	0.00191	0.01114	11.11.87 (54.1%)	20.10.87 (56.3%)	20.10.87 (56.3%)	20.10.87 (56.3%)
30.06.88	0.00163	0.00585	07.09.89 (87.8%)	16.10.89 (88.6%)	16.10.89 (88.6%)	16.10.89 (88.6%)
28.12.88	0.00064	0.00634	12.10.89 (89.4%)	07.09.89 (87.8%)	06.09.89 (88.1%)	07.09.89 (87.8%)
30.06.89	0.00039	0.00602	06.11.89 (91.1%)	12.10.89 (89.4%)	06.09.89 (88.1%)	30.08.89 (88.6%)
29.09.89	0.00045	0.00497	15.12.89 (98.3%)	11.12.89 (97.0%)	16.10.89 (88.6%)	12.10.89 (89.4%)



(a) $T =$ end of 1990



(b) $T =$ end of 1989



(c) $T =$ end of 1992

Figure 5. The results for the Nikkei index, base January 4, 1984: the enter (blue circles) and exit (red squares) dates for the values of the parameters $\alpha = 0.0$, $\mu_2 = -\mu_1$.

Table 5.: The exit dates for Nikkei stock average, $T =$ end of 1992

Enter	μ_1	σ	$\|\mu_2/\mu_1\|$			
			0.5	1.0	2.0	5.0
			$\alpha = 0.0$			
26.12.86	0.00086	0.01300	06.08.90 (73.5%)	02.04.90 (72.0%)	22.03.90 (76.7%)	19.03.90 (80.3%)
30.06.87	0.00191	0.01114	26.02.90 (85.6%)	11.11.87 (54.1%)	20.10.87 (56.3%)	20.10.87 (56.3%)
30.06.88	0.00163	0.00585	21.02.90 (91.8%)	21.02.90 (91.8%)	18.01.90 (94.4%)	16.01.90 (94.7%)
28.12.88	0.00064	0.00634	19.03.90 (80.3%)	13.03.90 (83.8%)	26.02.90 (85.6%)	23.02.90 (89.7%)
30.06.89	0.00039	0.00602	22.03.90 (76.7%)	14.03.90 (83.1%)	26.02.90 (85.6%)	23.02.90 (89.7%)
29.09.89	0.00045	0.00497	14.03.90 (83.1%)	26.02.90 (85.6%)	23.02.90 (89.7%)	23.02.90 (89.7%)
			$\alpha = -0.5$			
26.12.86	0.00086	0.01300	02.04.90 (72.0%)	02.04.90 (72.0%)	20.03.90 (79.2%)	11.11.87 (54.1%)
30.06.87	0.00191	0.01114	04.01.88 (54.5%)	10.11.87 (55.7%)	20.10.87 (56.3%)	20.10.87 (56.3%)
30.06.88	0.00163	0.00585	21.02.90 (91.8%)	21.02.90 (91.8%)	21.02.90 (91.8%)	16.01.90 (94.7%)
28.12.88	0.00064	0.00634	19.03.90 (80.3%)	26.02.90 (85.6%)	26.02.90 (85.6%)	23.02.90 (89.7%)
30.06.89	0.00039	0.00602	22.03.90 (76.7%)	14.03.90 (83.1%)	26.02.90 (85.6%)	23.02.90 (89.7%)
29.09.89	0.00045	0.00497	14.03.90 (83.1%)	26.02.90 (85.6%)	23.02.90 (89.7%)	21.02.90 (91.8%)
			$\alpha = -1.0$			
26.12.86	0.00086	0.01300	02.04.90 (72.0%)	22.03.90 (76.7%)	19.03.90 (80.3%)	11.11.87 (54.1%)
30.06.87	0.00191	0.01114	04.01.88 (54.5%)	10.11.87 (55.7%)	20.10.87 (56.3%)	20.10.87 (56.3%)
30.06.88	0.00163	0.00585	21.02.90 (91.8%)	21.02.90 (91.8%)	21.02.90 (91.8%)	16.01.90 (94.7%)
28.12.88	0.00064	0.00634	19.03.90 (80.3%)	26.02.90 (85.6%)	26.02.90 (85.6%)	23.02.90 (89.7%)
30.06.89	0.00039	0.00602	20.03.90 (79.2%)	13.03.90 (83.8%)	26.02.90 (85.6%)	23.02.90 (89.7%)
29.09.89	0.00045	0.00497	13.03.90 (83.8%)	26.02.90 (85.6%)	23.02.90 (89.7%)	21.02.90 (91.8%)

5. The Japanese golf course membership market

5.1 *Background*

In 1989 there were more than 400 golf courses in Japan with a total value more than US\$300 billion, a value larger than the Australian stock exchange capitalization of A\$250 billion. Memberships, which cost as much as US\$8 million, allowed play at a reduced cost plus the right to bring guests to play for a higher fee. However, their main value was not the ability to play golf but their share of the land occupied by the course and as an instrument to play the land market for relatively low stake with liquidity. These memberships were actively traded as speculative investments whose market was maintained by six market makers in Tokyo and Osaka. Weekly data was available in various areas of Japan since the beginning of 1982. This data was the best widely available data series on land prices in Japan and forms an ideal source for many types of analyses.

Rachev and Ziemba (1992) modeled the price changes as stable variants. The tails had considerable mass and the distributions were considered to have fat tails with a characteristic exponent about 1.4. This is consistent with the hypothesis that there was a speculative bubble in the late 1980s and the subsequent crash in 1990 to 1992.

Figure 6 shows the golf course membership GCM prices in various regions of the country: the western and the eastern parts of Japan, the Tokyo area and the nationwide average. The golf course memberships market was a much bigger bubble than the stock market.

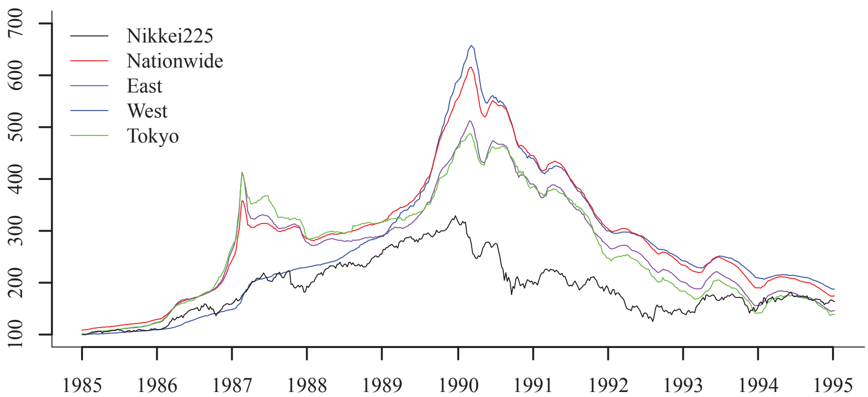


Figure 6. Graphs of the golf course membership prices in various areas of Japan and the Nikkei stock average, 1985-1995, with the 1985 values as 100%.

5.2 *Application of the changepoint detection method*

We applied the changepoint detection method to the golf prices for four different entering dates (July 1988, January 1989, July 1989, October 1989), four different values of the ratio μ_2/μ_1 (-0.5, -1.0, -2.0, -5.0) and the three utility functions $U_0(x) = \log x$, $U_{-1/2}(x) = -x^{-1/2}$, $U_{-1}(x) = -1/x$. In all the cases, $\sigma_2 = \sigma_1$. We use weekly data, and the parameters μ_1 and σ_1 are estimated using the 20 previous index weekly values before the enter date. Same as in the previous section we consider the time horizon T of the model corresponding to the end of 1989, 1990, and 1992.

The results are displayed in Tables 6 to 17 with the same notation as the previous section.

In all cases with T=1990, the model exits well above 90% of the global maximum price and all exits produced profits, see Figure 7. When T=1989, before the global peak in prices, the exits are near the top but not as high, about 88% of the peak with all the exits producing trading profits; see Figure 8. When T=1992, the results are similar to T=1990 as shown in Figure 9. The exits are very close (96-98%) to the peak and all entries had trading gains. See the discussion in section 6 about lags in prices with stocks leading golf course membership prices, leading all land declines in 1990-1991.

Remark. Compared with the stock market bubble, the changepoint model applied to the golf course membership index exits almost immediately after the peak. One reason for that is the higher signal-to-noise ratio μ_1/σ (see Tables 3–5 and 6–17): the statistic ψ_t increases "faster" when the drift switches to the value μ_2 from μ_1, as observed from the formula for ψ_t. Also we use *weekly* data for the golf market, so one period in changepoint detection delay in t-time corresponds to a week in calendar time, while for the stock market *daily* data is used.

Table 6.: Nationwide GCM prices, T = end of 1990

Enter	μ_1	σ	$\|\mu_2/\mu_1\|$ 0.5	1.0	2.0	5.0
			$\alpha = 0.0$			
26.06.88	0.00237	0.00280	25.03.90 (98.0%)	18.03.90 (98.9%)	18.03.90 (98.9%)	18.03.90 (98.9%)
25.12.88	0.00296	0.00228	18.03.90 (98.9%)	18.03.90 (98.9%)	18.03.90 (98.9%)	18.03.90 (98.9%)
25.06.89	0.00595	0.00347	18.03.90 (98.9%)	18.03.90 (98.9%)	18.03.90 (98.9%)	01.04.90 (95.0%)
24.09.89	0.01300	0.00689	25.03.90 (98.0%)	25.03.90 (98.0%)	25.03.90 (98.0%)	01.04.90 (95.0%)
			$\alpha = -0.5$			
26.06.88	0.00237	0.00280	25.03.90 (98.0%)	18.03.90 (98.9%)	18.03.90 (98.9%)	18.03.90 (98.9%)
25.12.88	0.00296	0.00228	18.03.90 (98.9%)	18.03.90 (98.9%)	18.03.90 (98.9%)	18.03.90 (98.9%)
25.06.89	0.00595	0.00347	18.03.90 (98.9%)	18.03.90 (98.9%)	18.03.90 (98.9%)	01.04.90 (95.0%)
24.09.89	0.01300	0.00689	25.03.90 (98.0%)	25.03.90 (98.0%)	25.03.90 (98.0%)	01.04.90 (95.0%)
			$\alpha = -1.0$			
26.06.88	0.00237	0.00280	25.03.90 (98.0%)	18.03.90 (98.9%)	18.03.90 (98.9%)	18.03.90 (98.9%)
25.12.88	0.00296	0.00228	18.03.90 (98.9%)	18.03.90 (98.9%)	18.03.90 (98.9%)	18.03.90 (98.9%)
25.06.89	0.00595	0.00347	18.03.90 (98.9%)	18.03.90 (98.9%)	18.03.90 (98.9%)	01.04.90 (95.0%)
24.09.89	0.01300	0.00689	18.03.90 (98.9%)	25.03.90 (98.0%)	25.03.90 (98.0%)	01.04.90 (95.0%)

Table 7.: East Japan GCM prices, T = end of 1990

Enter	μ_1	σ	$\|\mu_2/\mu_1\|$ 0.5	1.0	2.0	5.0
			$\alpha = 0.0$			
26.06.88	0.00165	0.00361	25.03.90 (96.7%)	25.03.90 (96.7%)	18.03.90 (98.2%)	18.03.90 (98.2%)
25.12.88	0.00182	0.00311	25.03.90 (96.7%)	18.03.90 (98.2%)	18.03.90 (98.2%)	18.03.90 (98.2%)
25.06.89	0.00446	0.00455	18.03.90 (98.2%)	18.03.90 (98.2%)	18.03.90 (98.2%)	18.03.90 (98.2%)
24.09.89	0.01191	0.00639	18.03.90 (98.2%)	18.03.90 (98.2%)	18.03.90 (98.2%)	01.04.90 (93.7%)
			$\alpha = -0.5$			
26.06.88	0.00165	0.00361	25.03.90 (96.7%)	25.03.90 (96.7%)	18.03.90 (98.2%)	18.03.90 (98.2%)
25.12.88	0.00182	0.00311	25.03.90 (96.7%)	18.03.90 (98.2%)	18.03.90 (98.2%)	18.03.90 (98.2%)
25.06.89	0.00446	0.00455	18.03.90 (98.2%)	18.03.90 (98.2%)	18.03.90 (98.2%)	18.03.90 (98.2%)
24.09.89	0.01191	0.00639	18.03.90 (98.2%)	18.03.90 (98.2%)	18.03.90 (98.2%)	01.04.90 (93.7%)
			$\alpha = -1.0$			
26.06.88	0.00165	0.00361	25.03.90 (96.7%)	25.03.90 (96.7%)	18.03.90 (98.2%)	18.03.90 (98.2%)
25.12.88	0.00182	0.00311	25.03.90 (96.7%)	18.03.90 (98.2%)	18.03.90 (98.2%)	18.03.90 (98.2%)
25.06.89	0.00446	0.00455	18.03.90 (98.2%)	18.03.90 (98.2%)	18.03.90 (98.2%)	18.03.90 (98.2%)
24.09.89	0.01191	0.00639	18.03.90 (98.2%)	18.03.90 (98.2%)	18.03.90 (98.2%)	01.04.90 (93.7%)

Table 8.: West Japan GCM prices, T = end of 1990

Enter	μ_1	σ	$\|\mu_2/\mu_1\|$ 0.5	1.0	2.0	5.0
			$\alpha = 0.0$			
26.06.88	0.00342	0.00235	25.03.90 (99.1%)	25.03.90 (99.1%)	25.03.90 (99.1%)	01.04.90 (96.4%)
25.12.88	0.00497	0.00349	25.03.90 (99.1%)	25.03.90 (99.1%)	01.04.90 (96.4%)	01.04.90 (96.4%)
25.06.89	0.00849	0.00715	01.04.90 (96.4%)	01.04.90 (96.4%)	01.04.90 (96.4%)	01.04.90 (96.4%)
24.09.89	0.01474	0.01141	01.04.90 (96.4%)	01.04.90 (96.4%)	01.04.90 (96.4%)	11.11.90 (69.6%)
			$\alpha = -0.5$			
26.06.88	0.00342	0.00235	25.03.90 (99.1%)	25.03.90 (99.1%)	25.03.90 (99.1%)	01.04.90 (96.4%)
25.12.88	0.00497	0.00349	25.03.90 (99.1%)	25.03.90 (99.1%)	01.04.90 (96.4%)	01.04.90 (96.4%)
25.06.89	0.00849	0.00715	01.04.90 (96.4%)	01.04.90 (96.4%)	01.04.90 (96.4%)	01.04.90 (96.4%)
24.09.89	0.01474	0.01141	01.04.90 (96.4%)	01.04.90 (96.4%)	01.04.90 (96.4%)	11.11.90 (69.6%)
			$\alpha = -1.0$			
26.06.88	0.00342	0.00235	25.03.90 (99.1%)	25.03.90 (99.1%)	25.03.90 (99.1%)	01.04.90 (96.4%)
25.12.88	0.00497	0.00349	25.03.90 (99.1%)	25.03.90 (99.1%)	01.04.90 (96.4%)	01.04.90 (96.4%)
25.06.89	0.00849	0.00715	01.04.90 (96.4%)	01.04.90 (96.4%)	01.04.90 (96.4%)	01.04.90 (96.4%)
24.09.89	0.01474	0.01141	01.04.90 (96.4%)	01.04.90 (96.4%)	01.04.90 (96.4%)	11.11.90 (69.6%)

Table 9.: Tokyo GCM prices, T = end of 1990

Enter	μ_1	σ	$\lvert\mu_2/\mu_1\rvert$			
			0.5	1.0	2.0	5.0
			$\alpha = 0.0$			
26.06.88	0.00173	0.00367	25.03.90 (96.5%)	18.03.90 (98.3%)	18.03.90 (98.3%)	09.04.89 (65.8%)
25.12.88	0.00278	0.00851	01.04.90 (94.1%)	01.04.90 (94.1%)	25.03.90 (96.5%)	25.03.90 (96.5%)
25.06.89	0.00189	0.00577	01.04.90 (94.1%)	25.03.90 (96.5%)	25.03.90 (96.5%)	18.03.90 (98.3%)
24.09.89	0.00759	0.00821	25.03.90 (96.5%)	25.03.90 (96.5%)	25.03.90 (96.5%)	01.04.90 (94.1%)
			$\alpha = -0.5$			
26.06.88	0.00173	0.00367	25.03.90 (96.5%)	18.03.90 (98.3%)	18.03.90 (98.3%)	09.04.89 (65.8%)
25.12.88	0.00278	0.00851	01.04.90 (94.1%)	01.04.90 (94.1%)	25.03.90 (96.5%)	25.03.90 (96.5%)
25.06.89	0.00189	0.00577	01.04.90 (94.1%)	25.03.90 (96.5%)	25.03.90 (96.5%)	18.03.90 (98.3%)
24.09.89	0.00759	0.00821	25.03.90 (96.5%)	25.03.90 (96.5%)	25.03.90 (96.5%)	01.04.90 (94.1%)
			$\alpha = -1.0$			
26.06.88	0.00173	0.00367	25.03.90 (96.5%)	18.03.90 (98.3%)	18.03.90 (98.3%)	09.04.89 (65.8%)
25.12.88	0.00278	0.00851	01.04.90 (94.1%)	01.04.90 (94.1%)	25.03.90 (96.5%)	25.03.90 (96.5%)
25.06.89	0.00189	0.00577	01.04.90 (94.1%)	25.03.90 (96.5%)	25.03.90 (96.5%)	18.03.90 (98.3%)
24.09.89	0.00759	0.00821	25.03.90 (96.5%)	25.03.90 (96.5%)	25.03.90 (96.5%)	01.04.90 (94.1%)

Table 10.: Nationwide GCM prices, T = end of 1989

Enter	μ_1	σ	$\lvert\mu_2/\mu_1\rvert$			
			0.5	1.0	2.0	5.0
			$\alpha = 0.0$			
26.06.88	0.00237	0.00280	17.12.89 (87.8%)	17.12.89 (87.8%)	03.12.89 (85.5%)	19.11.89 (83.0%)
25.12.88	0.00296	0.00228	17.12.89 (87.8%)	10.12.89 (86.5%)	03.12.89 (85.5%)	12.11.89 (82.2%)
25.06.89	0.00595	0.00347	17.12.89 (87.8%)	17.12.89 (87.8%)	10.12.89 (86.5%)	19.11.89 (83.0%)
24.09.89	0.01300	0.00689	17.12.89 (87.8%)	17.12.89 (87.8%)	03.12.89 (85.5%)	12.11.89 (82.2%)
			$\alpha = -0.5$			
26.06.88	0.00237	0.00280	17.12.89 (87.8%)	10.12.89 (86.5%)	03.12.89 (85.5%)	12.11.89 (82.2%)
25.12.88	0.00296	0.00228	17.12.89 (87.8%)	10.12.89 (86.5%)	03.12.89 (85.5%)	12.11.89 (82.2%)
25.06.89	0.00595	0.00347	17.12.89 (87.8%)	10.12.89 (86.5%)	03.12.89 (85.5%)	12.11.89 (82.2%)
24.09.89	0.01300	0.00689	17.12.89 (87.8%)	10.12.89 (86.5%)	03.12.89 (85.5%)	12.11.89 (82.2%)
			$\alpha = -1.0$			
26.06.88	0.00237	0.00280	17.12.89 (87.8%)	10.12.89 (86.5%)	03.12.89 (85.5%)	12.11.89 (82.2%)
25.12.88	0.00296	0.00228	17.12.89 (87.8%)	10.12.89 (86.5%)	03.12.89 (85.5%)	12.11.89 (82.2%)
25.06.89	0.00595	0.00347	17.12.89 (87.8%)	10.12.89 (86.5%)	03.12.89 (85.5%)	12.11.89 (82.2%)
24.09.89	0.01300	0.00689	17.12.89 (87.8%)	10.12.89 (86.5%)	03.12.89 (85.5%)	12.11.89 (82.2%)

Table 11.: East Japan GCM prices, T = end of 1989

Enter	μ_1	σ	$\lvert\mu_2/\mu_1\rvert$			
			0.5	1.0	2.0	5.0
			$\alpha = 0.0$			
26.06.88	0.00165	0.00361	17.12.89 (88.0%)	17.12.89 (88.0%)	10.12.89 (86.7%)	19.11.89 (84.3%)
25.12.88	0.00182	0.00311	17.12.89 (88.0%)	10.12.89 (86.7%)	03.12.89 (85.6%)	12.11.89 (83.7%)
25.06.89	0.00446	0.00455	17.12.89 (88.0%)	17.12.89 (88.0%)	10.12.89 (86.7%)	19.11.89 (84.3%)
24.09.89	0.01191	0.00639	17.12.89 (88.0%)	17.12.89 (88.0%)	10.12.89 (86.7%)	19.11.89 (84.3%)
			$\alpha = -0.5$			
26.06.88	0.00165	0.00361	17.12.89 (88.0%)	10.12.89 (86.7%)	03.12.89 (85.6%)	12.11.89 (83.7%)
25.12.88	0.00182	0.00311	17.12.89 (88.0%)	10.12.89 (86.7%)	03.12.89 (85.6%)	12.11.89 (83.7%)
25.06.89	0.00446	0.00455	17.12.89 (88.0%)	10.12.89 (86.7%)	03.12.89 (85.6%)	12.11.89 (83.7%)
24.09.89	0.01191	0.00639	17.12.89 (88.0%)	10.12.89 (86.7%)	03.12.89 (85.6%)	12.11.89 (83.7%)
			$\alpha = -1.0$			
26.06.88	0.00165	0.00361	17.12.89 (88.0%)	10.12.89 (86.7%)	03.12.89 (85.6%)	12.11.89 (83.7%)
25.12.88	0.00182	0.00311	17.12.89 (88.0%)	10.12.89 (86.7%)	03.12.89 (85.6%)	12.11.89 (83.7%)
25.06.89	0.00446	0.00455	17.12.89 (88.0%)	10.12.89 (86.7%)	03.12.89 (85.6%)	12.11.89 (83.7%)
24.09.89	0.01191	0.00639	17.12.89 (88.0%)	10.12.89 (86.7%)	03.12.89 (85.6%)	12.11.89 (83.7%)

Table 12.: West Japan GCM prices, T = end of 1989

Enter	μ_1	σ	$\lvert\mu_2/\mu_1\rvert$ 0.5	1.0	2.0	5.0
			$\alpha = 0.0$			
26.06.88	0.00342	0.00235	24.12.89 (88.8%)	17.12.89 (87.0%)	10.12.89 (85.7%)	19.11.89 (81.0%)
25.12.88	0.00497	0.00349	17.12.89 (87.0%)	10.12.89 (85.7%)	03.12.89 (85.0%)	12.11.89 (79.9%)
25.06.89	0.00849	0.00715	17.12.89 (87.0%)	17.12.89 (87.0%)	03.12.89 (85.0%)	19.11.89 (81.0%)
24.09.89	0.01474	0.01141	17.12.89 (87.0%)	10.12.89 (85.7%)	03.12.89 (85.0%)	12.11.89 (79.9%)
			$\alpha = -0.5$			
26.06.88	0.00342	0.00235	17.12.89 (87.0%)	10.12.89 (85.7%)	03.12.89 (85.0%)	12.11.89 (79.9%)
25.12.88	0.00497	0.00349	17.12.89 (87.0%)	10.12.89 (85.7%)	03.12.89 (85.0%)	12.11.89 (79.9%)
25.06.89	0.00849	0.00715	17.12.89 (87.0%)	10.12.89 (85.7%)	03.12.89 (85.0%)	12.11.89 (79.9%)
24.09.89	0.01474	0.01141	17.12.89 (87.0%)	10.12.89 (85.7%)	03.12.89 (85.0%)	12.11.89 (79.9%)
			$\alpha = -1.0$			
26.06.88	0.00342	0.00235	17.12.89 (87.0%)	10.12.89 (85.7%)	03.12.89 (85.0%)	12.11.89 (79.9%)
25.12.88	0.00497	0.00349	17.12.89 (87.0%)	10.12.89 (85.7%)	03.12.89 (85.0%)	12.11.89 (79.9%)
25.06.89	0.00849	0.00715	17.12.89 (87.0%)	10.12.89 (85.7%)	03.12.89 (85.0%)	12.11.89 (79.9%)
24.09.89	0.01474	0.01141	17.12.89 (87.0%)	10.12.89 (85.7%)	03.12.89 (85.0%)	12.11.89 (79.9%)

Table 13.: Tokyo GCM prices, T = end of 1989

Enter	μ_1	σ	$\lvert\mu_2/\mu_1\rvert$ 0.5	1.0	2.0	5.0
			$\alpha = 0.0$			
26.06.88	0.00173	0.00367	17.12.89 (91.3%)	17.12.89 (91.3%)	09.04.89 (65.8%)	09.04.89 (65.8%)
25.12.88	0.00278	0.00851	10.12.89 (89.0%)	10.12.89 (89.0%)	03.12.89 (88.2%)	12.11.89 (83.9%)
25.06.89	0.00189	0.00577	17.12.89 (91.3%)	10.12.89 (89.0%)	03.12.89 (88.2%)	12.11.89 (83.9%)
24.09.89	0.00759	0.00821	17.12.89 (91.3%)	10.12.89 (89.0%)	10.12.89 (89.0%)	12.11.89 (83.9%)
			$\alpha = -0.5$			
26.06.88	0.00173	0.00367	17.12.89 (91.3%)	10.12.89 (89.0%)	09.04.89 (65.8%)	09.04.89 (65.8%)
25.12.88	0.00278	0.00851	10.12.89 (89.0%)	10.12.89 (89.0%)	03.12.89 (88.2%)	12.11.89 (83.9%)
25.06.89	0.00189	0.00577	17.12.89 (91.3%)	10.12.89 (89.0%)	03.12.89 (88.2%)	12.11.89 (83.9%)
24.09.89	0.00759	0.00821	17.12.89 (91.3%)	10.12.89 (89.0%)	03.12.89 (88.2%)	12.11.89 (83.9%)
			$\alpha = -1.0$			
26.06.88	0.00173	0.00367	17.12.89 (91.3%)	10.12.89 (89.0%)	09.04.89 (65.8%)	09.04.89 (65.8%)
25.12.88	0.00278	0.00851	10.12.89 (89.0%)	10.12.89 (89.0%)	03.12.89 (88.2%)	12.11.89 (83.9%)
25.06.89	0.00189	0.00577	17.12.89 (91.3%)	10.12.89 (89.0%)	03.12.89 (88.2%)	12.11.89 (83.9%)
24.09.89	0.00759	0.00821	17.12.89 (91.3%)	10.12.89 (89.0%)	03.12.89 (88.2%)	12.11.89 (83.9%)

Table 14.: Nationwide GCM prices, T = end of 1992

Enter	μ_1	σ	$\lvert\mu_2/\mu_1\rvert$ 0.5	1.0	2.0	5.0
			$\alpha = 0.0$			
26.06.88	0.00237	0.00280	25.03.90 (98.0%)	25.03.90 (98.0%)	18.03.90 (98.9%)	18.03.90 (98.9%)
25.12.88	0.00296	0.00228	18.03.90 (98.9%)	18.03.90 (98.9%)	18.03.90 (98.9%)	18.03.90 (98.9%)
25.06.89	0.00595	0.00347	18.03.90 (98.9%)	18.03.90 (98.9%)	18.03.90 (98.9%)	01.04.90 (95.0%)
24.09.89	0.01300	0.00689	25.03.90 (98.0%)	25.03.90 (98.0%)	01.04.90 (95.0%)	01.04.90 (95.0%)
			$\alpha = -0.5$			
26.06.88	0.00237	0.00280	25.03.90 (98.0%)	25.03.90 (98.0%)	18.03.90 (98.9%)	18.03.90 (98.9%)
25.12.88	0.00296	0.00228	18.03.90 (98.9%)	18.03.90 (98.9%)	18.03.90 (98.9%)	18.03.90 (98.9%)
25.06.89	0.00595	0.00347	18.03.90 (98.9%)	18.03.90 (98.9%)	18.03.90 (98.9%)	01.04.90 (95.0%)
24.09.89	0.01300	0.00689	25.03.90 (98.0%)	25.03.90 (98.0%)	01.04.90 (95.0%)	01.04.90 (95.0%)
			$\alpha = -1.0$			
26.06.88	0.00237	0.00280	25.03.90 (98.0%)	25.03.90 (98.0%)	18.03.90 (98.9%)	18.03.90 (98.9%)
25.12.88	0.00296	0.00228	18.03.90 (98.9%)	18.03.90 (98.9%)	18.03.90 (98.9%)	18.03.90 (98.9%)
25.06.89	0.00595	0.00347	18.03.90 (98.9%)	18.03.90 (98.9%)	18.03.90 (98.9%)	01.04.90 (95.0%)
24.09.89	0.01300	0.00689	25.03.90 (98.0%)	25.03.90 (98.0%)	01.04.90 (95.0%)	01.04.90 (95.0%)

Table 15.: East Japan GCM prices, T = end of 1992

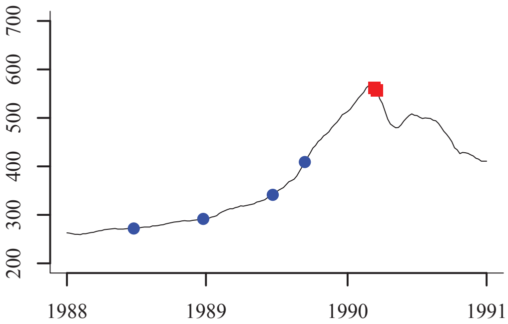
Enter	μ_1	σ	$\|\mu_2/\mu_1\|$			
			0.5	1.0	2.0	5.0
			$\alpha = 0.0$			
26.06.88	0.00165	0.00361	01.04.90 (93.7%)	25.03.90 (96.7%)	25.03.90 (96.7%)	18.03.90 (98.2%)
25.12.88	0.00182	0.00311	25.03.90 (96.7%)	25.03.90 (96.7%)	18.03.90 (98.2%)	18.03.90 (98.2%)
25.06.89	0.00446	0.00455	25.03.90 (96.7%)	25.03.90 (96.7%)	18.03.90 (98.2%)	25.03.90 (96.7%)
24.09.89	0.01191	0.00639	18.03.90 (98.2%)	18.03.90 (98.2%)	25.03.90 (96.7%)	01.04.90 (93.7%)
			$\alpha = -0.5$			
26.06.88	0.00165	0.00361	01.04.90 (93.7%)	25.03.90 (96.7%)	25.03.90 (96.7%)	18.03.90 (98.2%)
25.12.88	0.00182	0.00311	25.03.90 (96.7%)	25.03.90 (96.7%)	18.03.90 (98.2%)	18.03.90 (98.2%)
25.06.89	0.00446	0.00455	25.03.90 (96.7%)	25.03.90 (96.7%)	18.03.90 (98.2%)	25.03.90 (96.7%)
24.09.89	0.01191	0.00639	18.03.90 (98.2%)	18.03.90 (98.2%)	25.03.90 (96.7%)	01.04.90 (93.7%)
			$\alpha = -1.0$			
26.06.88	0.00165	0.00361	01.04.90 (93.7%)	25.03.90 (96.7%)	25.03.90 (96.7%)	18.03.90 (98.2%)
25.12.88	0.00182	0.00311	25.03.90 (96.7%)	25.03.90 (96.7%)	18.03.90 (98.2%)	18.03.90 (98.2%)
25.06.89	0.00446	0.00455	25.03.90 (96.7%)	25.03.90 (96.7%)	18.03.90 (98.2%)	25.03.90 (96.7%)
24.09.89	0.01191	0.00639	18.03.90 (98.2%)	18.03.90 (98.2%)	25.03.90 (96.7%)	01.04.90 (93.7%)

Table 16.: West Japan GCM prices, T = end of 1992

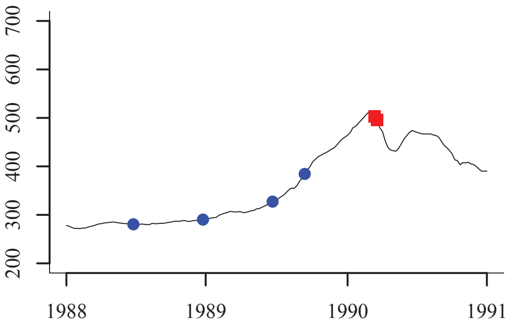
Enter	μ_1	σ	$\|\mu_2/\mu_1\|$			
			0.5	1.0	2.0	5.0
			$\alpha = 0.0$			
26.06.88	0.00342	0.00235	25.03.90 (99.1%)	25.03.90 (99.1%)	25.03.90 (99.1%)	01.04.90 (96.4%)
25.12.88	0.00497	0.00349	01.04.90 (96.4%)	01.04.90 (96.4%)	01.04.90 (96.4%)	01.04.90 (96.4%)
25.06.89	0.00849	0.00715	01.04.90 (96.4%)	01.04.90 (96.4%)	01.04.90 (96.4%)	01.04.90 (96.4%)
24.09.89	0.01474	0.01141	08.04.90 (94.5%)	08.04.90 (94.5%)	08.04.90 (94.5%)	08.11.92 (38.9%)
			$\alpha = -0.5$			
26.06.88	0.00342	0.00235	25.03.90 (99.1%)	25.03.90 (99.1%)	25.03.90 (99.1%)	01.04.90 (96.4%)
25.12.88	0.00497	0.00349	01.04.90 (96.4%)	01.04.90 (96.4%)	01.04.90 (96.4%)	01.04.90 (96.4%)
25.06.89	0.00849	0.00715	01.04.90 (96.4%)	01.04.90 (96.4%)	01.04.90 (96.4%)	01.04.90 (96.4%)
24.09.89	0.01474	0.01141	08.04.90 (94.5%)	01.04.90 (96.4%)	08.04.90 (94.5%)	08.11.92 (38.9%)
			$\alpha = -1.0$			
26.06.88	0.00342	0.00235	25.03.90 (99.1%)	25.03.90 (99.1%)	25.03.90 (99.1%)	01.04.90 (96.4%)
25.12.88	0.00497	0.00349	01.04.90 (96.4%)	01.04.90 (96.4%)	01.04.90 (96.4%)	01.04.90 (96.4%)
25.06.89	0.00849	0.00715	01.04.90 (96.4%)	01.04.90 (96.4%)	01.04.90 (96.4%)	01.04.90 (96.4%)
24.09.89	0.01474	0.01141	01.04.90 (96.4%)	01.04.90 (96.4%)	08.04.90 (94.5%)	08.11.92 (38.9%)

Table 17.: Tokyo GCM prices, T = end of 1992

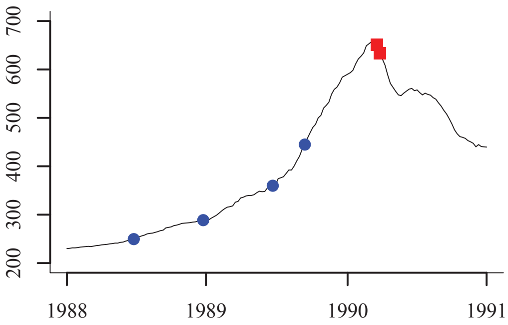
Enter	μ_1	σ	$\|\mu_2/\mu_1\|$			
			0.5	1.0	2.0	5.0
			$\alpha = 0.0$			
26.06.88	0.00173	0.00367	01.04.90 (94.1%)	25.03.90 (96.5%)	25.03.90 (96.5%)	09.04.89 (65.8%)
25.12.88	0.00278	0.00851	15.04.90 (89.7%)	08.04.90 (92.2%)	01.04.90 (94.1%)	01.04.90 (94.1%)
25.06.89	0.00189	0.00577	08.04.90 (92.2%)	01.04.90 (94.1%)	01.04.90 (94.1%)	25.03.90 (96.5%)
24.09.89	0.00759	0.00821	01.04.90 (94.1%)	25.03.90 (96.5%)	25.03.90 (96.5%)	01.04.90 (94.1%)
			$\alpha = -0.5$			
26.06.88	0.00173	0.00367	01.04.90 (94.1%)	25.03.90 (96.5%)	25.03.90 (96.5%)	09.04.89 (65.8%)
25.12.88	0.00278	0.00851	15.04.90 (89.7%)	08.04.90 (92.2%)	01.04.90 (94.1%)	01.04.90 (94.1%)
25.06.89	0.00189	0.00577	08.04.90 (92.2%)	01.04.90 (94.1%)	01.04.90 (94.1%)	25.03.90 (96.5%)
24.09.89	0.00759	0.00821	01.04.90 (94.1%)	25.03.90 (96.5%)	25.03.90 (96.5%)	01.04.90 (94.1%)
			$\alpha = -1.0$			
26.06.88	0.00173	0.00367	01.04.90 (94.1%)	25.03.90 (96.5%)	25.03.90 (96.5%)	09.04.89 (65.8%)
25.12.88	0.00278	0.00851	15.04.90 (89.7%)	08.04.90 (92.2%)	01.04.90 (94.1%)	01.04.90 (94.1%)
25.06.89	0.00189	0.00577	08.04.90 (92.2%)	01.04.90 (94.1%)	01.04.90 (94.1%)	25.03.90 (96.5%)
24.09.89	0.00759	0.00821	01.04.90 (94.1%)	25.03.90 (96.5%)	25.03.90 (96.5%)	01.04.90 (94.1%)



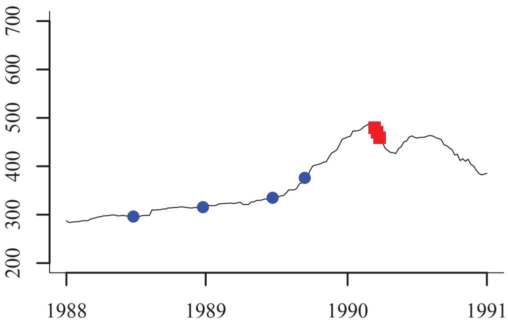
(a) Nationwide GCM



(b) East Japan GCM

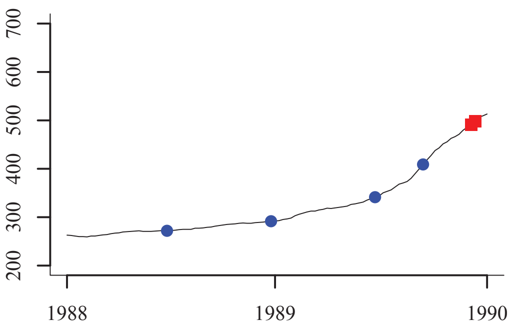


(c) West Japan GCM

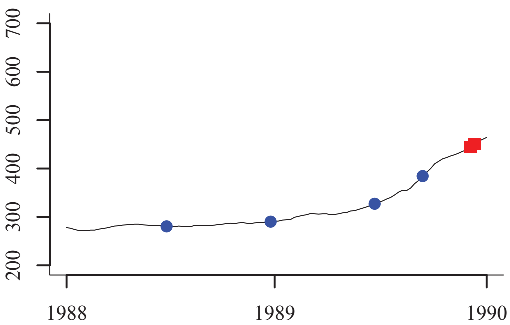


(d) Tokyo GCM

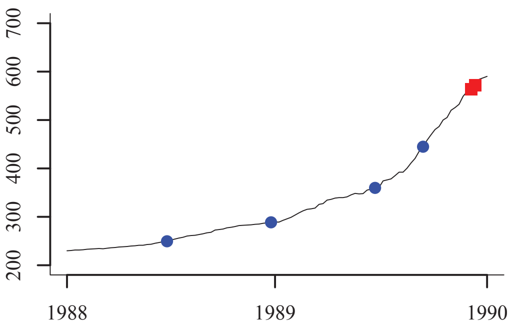
Figure 7. The results for the GCM prices when T = end of 1990: the enter (blue circles) and exit (red squares) dates for the values of the parameters $\alpha = 0.0$, $\mu_2 = -\mu_1$.



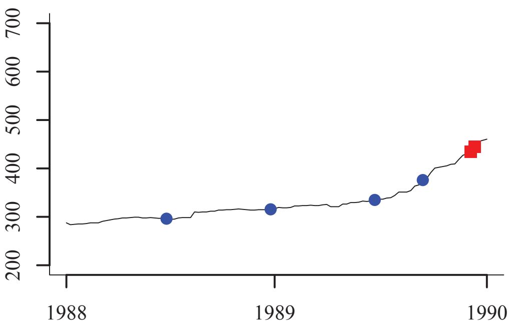
(a) Nationwide GCM



(b) East Japan GCM

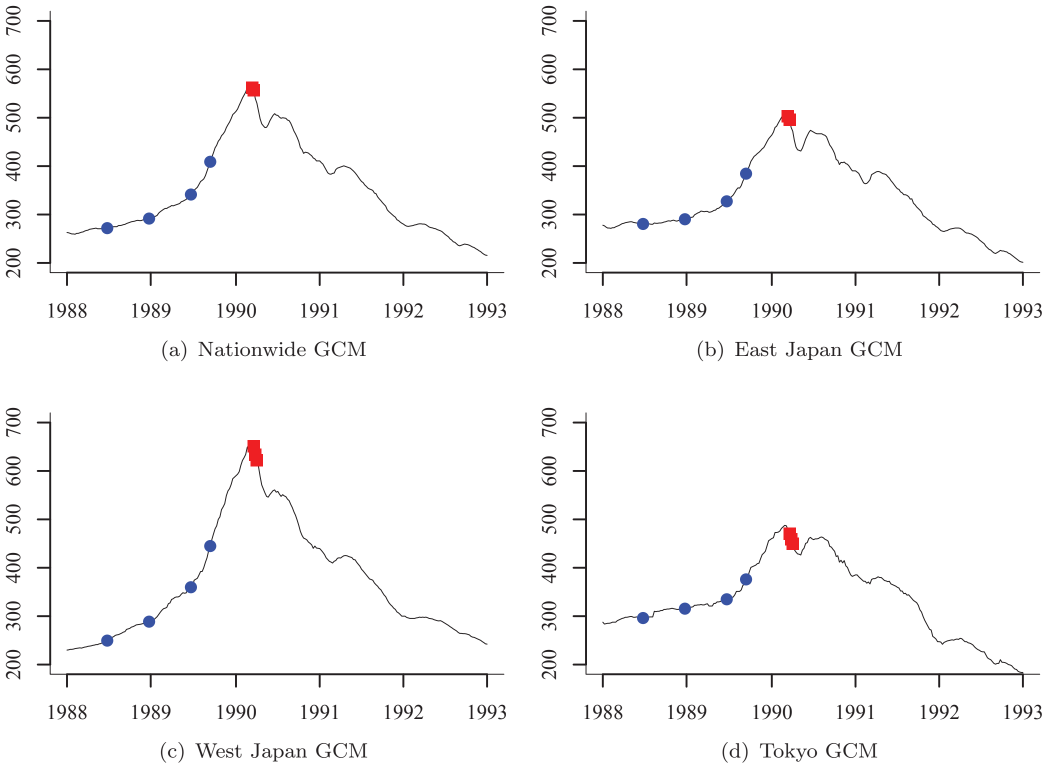


(c) West Japan GCM



(d) Tokyo GCM

Figure 8. The results for the GCM prices when T = end of 1989: the enter (blue circles) and exit (red squares) dates for the values of the parameters $\alpha = 0.0$, $\mu_2 = -\mu_1$.



(a) Nationwide GCM

(b) East Japan GCM

(c) West Japan GCM

(d) Tokyo GCM

Figure 9. The results for the GCM prices when T = end of 1992: the enter (blue circles) and exit (red squares) dates for the values of the parameters $\alpha = 0.0$, $\mu_2 = -\mu_1$.

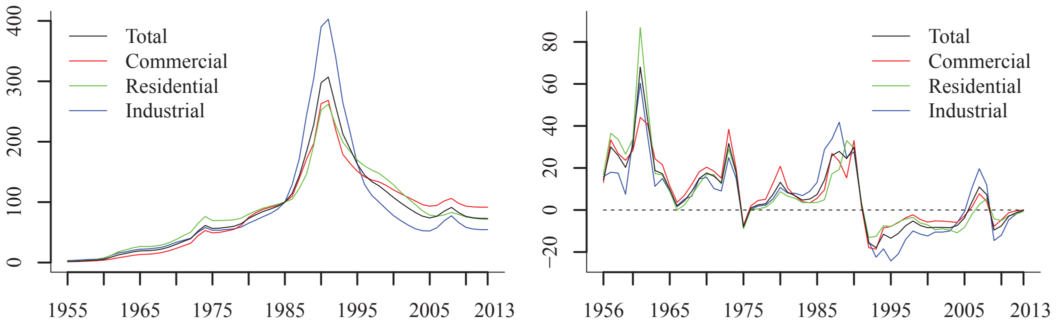
6. The overall Japanese land market, 1955 to 2013

Figures 10 and 11 and Table 18 give the Japan Real Estate Institute's land indices for the six largest cities, and for all of Japan for commercial, housing, industrial and total land for each six month period from 1955 to 2013. The figures also gives the yearly rate of changes. The six largest cities are Tokyo, Yokohama, Osaka, Nagoya, Kobe and Kyoto.

The country wide indices are based on 140 cities. The data are appraisal based which tends to smooth the price levels and lag the market. Simple averages of samples of ten lots in each city form the indices which were normalized at 100 as of 1985. The sampling procedure separates land into high, medium and low grades reflecting location, social circumstances, yield, etc. The sampling procedure selects lots randomly and equally from each of these three classes.

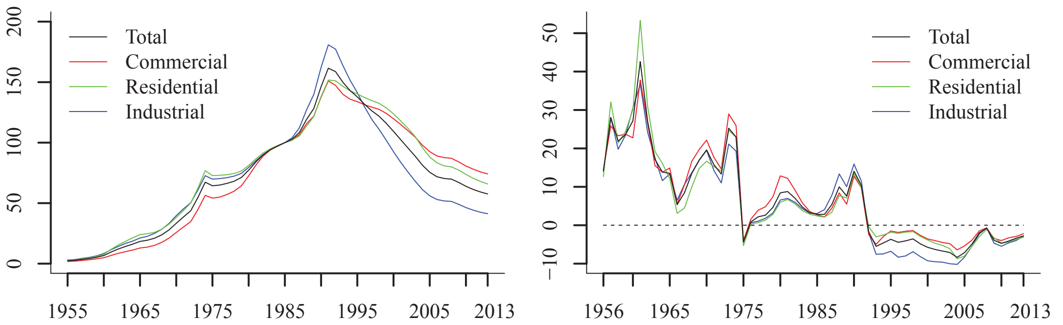
Table 18 also indicates that the price increase has been largest in the six largest cities. Despite large rises in the 1980s, the relative gain in the period 1955 to 1970 was much larger than from 1970 to the circa 1990 peak. For land in the whole country, the 1955 to 1970 period produced gains of about 15 times the 1955 values. These prices then increased only about four fold in the ensuing twenty years. In the six largest cities, the increase was also much larger in the 1955 to 1970 period versus the next two decades.

Land values in the six largest cities outpaced the Consumer Prices Index (CPI) by twenty times from 1955 to 1990. In the Ginza district of Tokyo each square meter of land was worth well over US\$200,000 with some plots approaching \$300,000. Choice downtown land in Tokyo sold for the equivalent of nearly a billion dollars an acre. At neighboring land prices, the value of land under the Emperor's palace and garden in Tokyo equaled that of all California or of Canada. The total land value in Japan in 1990 was about 4.1 times that of the whole United States. Japanese land was worth some ¥2180 trillion as of the end of 1989. This compares with a value of ¥1050 trillion at the end of 1985. Using an exchange rate of ¥143.76 per dollar at the end of 1989, gave a land value of \$15.16 trillion. As of September 1990, all land had an index of



(a) In the six largest cities (b) Average annual rates of price change, % (six cities)

Figure 10. Land price indices for industrial, residential, commercial and all land and annual rates of price change for six largest cities, 1955 to 2013. Source: Japan Real Estate Institute



(a) In the six largest cities (b) Average annual rates of price change, % (six cities)

Figure 11. Land price indices for industrial, residential, commercial and all land and annual rates of price change for all land, 1955 to 2013. Source: Japan Real Estate Institute

203.1, up 16.2% from September 1989.

With an end of 1990 exchange rate of 135.40, total Japanese land values were in the $18.7 trillion range in late 1990. The average acre of land in Japan was worth fully 100 times the average acre in the U.S. So even though the US has about 25 times more land than Japan, its 1990 total value was less than a fourth as much. Essentially half the world's land value at 1987-90 prices was accounted for by Japanese land! It also accounted for about 20% of the total non-human capital asset value in the world. Simple houses in Tokyo rented for more than $10,000 per month and cost in the millions. Office space for sale in Tokyo's financial district cost nearly $75,000 per square foot. Some luxury apartments in Tokyo rented for well over $20,000 per month.

Table 18. Increase of land prices, 1955 to 1990, %. Source: Japan Real Estate Institute

	Nationwide				6 largest cities			
	Total	Commercial	Residential	Industrial	Total	Commercial	Residential	Industrial
1955 to 1990	65.5	59.6	81.2	56.7	178.8	127.7	219.1	150.8
1955 to 1970	15.1	14.5	15.5	15.8	18.7	11.0	20.2	23.6
1970 to 1990	4.3	4.1	5.2	3.6	9.6	11.6	10.8	6.4

In 1988 Tokyo's land value alone was about $7.7 trillion, or about half the land value of the whole country. To understand how much this is we can do a idealized experiment. Let's borrow on it up to 80% of its value. Banks in Tokyo commonly provided such loans based on land security until the high interest rates of late 1989 into 1990. From 1987 to 1989, the interest rates on loans secured by land were 5.7% and 6.6% for variable and fixed rate loans, respectively. We would then have almost enough money to purchase all the land in the U.S. for $3.7 trillion and all the stock on the New York, American and NASDAQ over-the-counter stock exchanges for about \$2.6 trillion in an all-cash transaction. Obviously, one could not sell all of Tokyo's land for \$7.7 trillion quickly, nor would a group of banks undertake such a large loan, but this was the value of land prices in fiscal 1987. In Tokyo about 2% of land changed hands each year. The price was kept up and bid higher because of the excess of demand over supply.

A staggering 56% of the national wealth of Japan was land. The 1990 percentage was even higher since there was a huge price increase in 1986 and steady rises since then. Land turnover was very small as the Japanese believe in holding land whenever possible. This was reinforced by the tax system which encourages the purchase of more land and discourages land sales. The population in per unit of habitable area was thirty times higher in Japan than the US. The GNP and energy consumption per habitable area were also much higher in Japan than in the U.S. (though the energy per unit GNP was much lower in Japan), see Table 19. This put upward pressure on land prices.

Table 19. Comparison of Fundamentals, Japan and the U.S., 1989. Source: Daiwa Securities America, Inc

	Japan	US	Japan as % of US
Population, millions	120	239	50.21
Total area (1000 sq km)	377	9373	4.02
Habitable area (1000 sq km)	80	4786	1.67
Population per habitable area (pop/sq km)	1500	50	3,000.00
GNP per habitable area (million \$/sq km)	16.90	0.80	2,122.50
Energy consumption (tone oil equivalent/sq km)	4650	390	1,192.38

Boone (1989) developed several models in an attempt to rationalize the high land values in Japan from an economic point of view. He found that if Japan's GNP growth exceeded that in the U.S. by about 2% per year forever, then land prices 100 times higher in Japan than in the U.S. are consistent with the economic model.

High interest rates which led to a sharp fall in stock prices in 1990 did not lead to any decline in land prices until 1991 as shown in Figures 10 and 11. However, there was a sharp decline in speculative land such as golf course membership and condos, see Stone and Ziemba (1993). As interest rates rise, land demand fell but in Tokyo, with virtually no new supply, demand still greatly exceeded supply. At the same time supply declines with higher interest rates as development costs are curtailed. All the incentives favored holding land and not even developing it. As Canaway (1990) pointed out, land held less than five years was taxed at fully 52% of its sale value. Meanwhile, yearly taxes paid to hold land were about 0.05 to 0.10% of its current value. Even upon death it paid to borrow money which was deductible at full value while land was valued at about half its market value. Hence inheritance taxes are minimized. Canaway argued that in a major crash the stock market will go first, then the economy and finally the land markets. Our results confirm this.

7. Applying the model to the Nikkei in 2013

In 2013 a policy to devalue the usually strong yen led to a large increase in stock prices. Since the -48% crash in 1990, the Nikkei has had its ups and downs but, as shown in Figure 4, it has never really recovered to the 1990 peak.

We apply the changepoint detection model to the Nikkei225 with T corresponding to the end of 2013. Table 20 shows the exits from the market past the 2013 local peak, and Figure 12 illustrates the results for the parameters $\alpha = 0$, $\mu_2 = -\mu_1$, using data to October 2013. All the entries have exits with profits.

Table 20.: Nikkei-225 from November 2012 to October 2013

Enter	μ_1	σ	$\lvert\mu_2/\mu_1\rvert$ 0.5	1.0	2.0	5.0
			$\alpha = 0.0$			
01.02.13	0.00512	0.01206	30.05.13 (87.0%)	30.05.13 (87.0%)	23.05.13 (92.7%)	23.05.13 (92.7%)
01.03.13	0.00383	0.01527	13.06.13 (79.6%)	03.06.13 (84.9%)	30.05.13 (87.0%)	23.05.13 (92.7%)
01.04.13	0.00269	0.01596	22.08.13 (85.5%)	13.06.13 (79.6%)	03.06.13 (84.9%)	30.05.13 (87.0%)
			$\alpha = -0.5$			
01.02.13	0.00512	0.01206	30.05.13 (87.0%)	27.05.13 (90.5%)	23.05.13 (92.7%)	23.05.13 (92.7%)
01.03.13	0.00383	0.01527	13.06.13 (79.6%)	03.06.13 (84.9%)	30.05.13 (87.0%)	23.05.13 (92.7%)
01.04.13	0.00269	0.01596	20.08.13 (85.7%)	07.06.13 (82.4%)	03.06.13 (84.9%)	30.05.13 (87.0%)
			$\alpha = -1.0$			
01.02.13	0.00512	0.01206	30.05.13 (87.0%)	27.05.13 (90.5%)	23.05.13 (92.7%)	23.05.13 (92.7%)
01.03.13	0.00383	0.01527	06.06.13 (82.6%)	03.06.13 (84.9%)	30.05.13 (87.0%)	23.05.13 (92.7%)
01.04.13	0.00269	0.01596	20.08.13 (85.7%)	06.06.13 (82.6%)	03.06.13 (84.9%)	30.05.13 (87.0%)

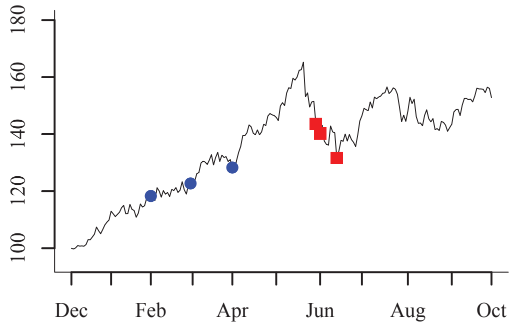


Figure 12. Nikkei225 from November 2012 to October 2013.

8. Short selling the Nikkei portfolio

Would it be possible to apply the changepoint detection theory to profit from the price decline by short selling the index portfolio when the changepoint is detected?

Consider the method which consists in selling short the index portfolio when the crash of the bubble is detected, and finding the stopping time τ to buy the portfolio back which *minimizes* $\mathsf{E}\log(S_\tau)$[1]. The latter problem is equivalent to maximizing $\mathsf{E}\log(1/S_\tau)$ and reduces to Theorem 3.1 with $\tilde{S}_t = 1/S_t$. We applied this method to the Nikkei index portfolio in 1990 and 2013 with short selling on 16.01.90, 21.02.90, 23.02.90, and 30.05.13, 03.06.13, which are the dates when the crashes are detected by the changepoint model with parameters $\alpha = 0$ and $\mu_2 = -\mu_1$ (see Tables 3 and 20). In the problem of minimizing $\mathsf{E}\log(S_\tau)$, we set $\mu_1 = -\hat{\mu}$, $\sigma = \hat{\sigma}$, where $\hat{\mu}$ and $\hat{\sigma}$ are the estimated parameters of the drift and volatility of the log-prices during the 100 trading days before the market peaks on December 29, 1989 and March 22, 2013. The value of μ_2 varies through $\mu_2 = -0.5\mu_1, -\mu_1, -2\mu_1, -3\mu_1, -5\mu_1$, and T corresponds to the end of 1990 and 2013 respectively.

The exit dates obtained for both 1990 and 2013 do not depend on the entry dates. For 1990, we get 11.05.90 for $\mu_2 = -0.5\mu_1$, 07.05.90 for $\mu_2 = -\mu_1$, 09.04.90 for $\mu_2 = -2\mu_1$ and 26.03.90 for $\mu_2 = -3\mu_1$ and $\mu_2 = -5\mu_1$. The corresponding daily closing prices are 31512, 30956, 30398 and 31840, while the entry prices are 36850 (on 16.01.90), 35734 (21.02.90) and 34891 (23.02.90); the local trough was on April 2, 1990 at 28002. Thus the strategy was profitable in this case.

In 2013, for the both entry dates the exits are 05.07.13 for $\mu_2 = -0.5\mu_1$, 02.07.90 for $\mu_2 = -\mu_1$, $\mu_2 = -\mu_1, -2\mu_1, -3\mu_1$, and 01.07.13 for $\mu_2 = -5\mu_1$. The corresponding daily closing prices, 14309.97, 14098.74, and 13852.50 are close to the local minimum at 12445.38 (June 13, 2013), however the strategy was unprofitable.

[1]Other utility functions can be considered in a similar way.

One can use these results to develop a multi-changepoint detection model, which buys (sells short) assets when the signal-to-noise ratio is high and positive (respectively, negative with large absolute value) and sells (buys) after it detects a changepoint, repeating this procedure continuously in time. We leave the analysis of this strategy for future research. The substantial difficulty here consist in the estimation of the risk of such a strategy, since our assumptions (mainly, log-normal returns with constant parameters, and the loose method of choosing T) do not allow an accurate quantitative prediction of the strategy performance. A theoretical result for a multiple changepoint model for a Brownian motion, which is a continuous-time analogue of our model, was obtained by Gapeev (2010).

9. Conclusion

This paper studies the Nikkei stock average and golf course membership prices around 1990 in Japan. Both of these asset markets were in bubble type markets and had huge rises up and then dramatic falls. High interest rates relative to stock earnings seems to be the main cause of both crashes. The bond stock earnings yield difference model predicted the stock market crash. In time sequence, the stock market fell first then the golf course membership market and finally the land markets. We use a changepoint detection model designed to exit bubble type markets and apply it successfully to the Nikkei stock average and the golf course membership markets in various areas of Japan.

Data from 1990 to 2013 indicate that both the stock and land markets have never recovered to their circa 1990 prices. The Nikkei had a big rise in 2013 fueled by a weak yen policy and this market peaked in mid 2013 and the changepoint detection model suggested an exit then. In virtually all entry cases the stopping rule model has good exits as they were close to the global price peaks and produce profits.

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