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Green Capital Requirements

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Abstract

We study bank capital requirements as a tool to address financial risks and externalities caused by carbon emissions. Capital regulation can effectively address financial risks but doing so does not necessarily reduce emissions (e.g., higher capital requirements for carbon-intensive loans may crowd out clean lending). Reducing emissions via capital requirements may require sacrificing financial stability or may be altogether infeasible. Carbon taxes are not subject to these drawbacks. However, if the government cannot commit to future environmental policies, capital requirements can make higher carbon taxes credible by ensuring banks have sufficient capital to absorb losses from stranded asset risk.

Keywords: Bank Capital Regulation, Capital Requirements, Climate Change, Climate Risk, Transition Risks, Physical Risks, Stranded Assets, Green Supporting Factor, Brown Penalizing Factor.

JEL Classification: G21, G28

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Climate change has become the topic of an active policy debate at central banks and financial regulators.[1] From the perspective of bank regulators, climate change is potentially relevant along two dimensions. First, as a consequence of climate change, the banking sector could be exposed to financial risks that are not captured by the current regulatory framework. Second, in the absence of a global carbon tax, some policymakers have argued that capital requirements could serve as a means to reduce carbon externalities.[2]

To investigate these issues, we embed climate-related risks into an otherwise standard model of bank capital regulation. Our positive results show that the effects of green tilts to capital regulation on credit allocation can be subtle. For example, higher capital requirements for dirty loans can reduce clean lending. Conversely, decreases in capital requirements for clean loans can increase dirty lending. These results obtain because changes in capital requirements affect credit allocation via the marginal loan, which can be clean or dirty.

From a normative perspective, our analysis shows that capital requirements can be an effective tool to deal with prudential risks arising from climate change. However, addressing climate-related financial risks via capital requirements is not equivalent to reducing emissions. For example, it can be optimal for a prudential regulator to increase capital requirements on loans affected by climate risk even if this crowds out clean lending. In contrast, using capital requirements to discourage the funding of carbon-intensive activities is less promising. When bank capital is ample, capital regulation is powerless to deter the funding of financially profitable dirty loans even if they generate negative social value. When bank capital is scarce, inducing banks not to fund dirty loans can require lowering capital requirements for clean loans below the prudentially optimal level, thereby sacrificing financial stability. In addition, even if capital regulation can successfully remove dirty loans from the banking system, high-emitting activities will likely attract funding elsewhere as long as they offer a positive return to investors.

We conclude that interventions that directly reduce the profitability of carbon-intensive investments (e.g., a carbon tax) are a more effective way to reduce carbon emissions. In this context, capital requirements can play an indirect role: By ensuring sufficient loss-absorbing capital in the banking sector, they can help facilitate the introduction of carbon taxes or other measures, which governments may be reluctant to introduce as long as the resulting revaluation of bank assets ("stranded assets") could trigger a banking crisis.

We develop these insights in the context of a model in which banks extend loans to

[1] See, e.g., van Steenis (2019), ECB (2021), and Financial Stability Board (2022).
[2] See Dombrovskis (2017).

two types of borrowers, dirty (high carbon emissions) and clean (no emissions). Loans to both types of firms are risky and, when banks cannot repay deposits in full, the deposit insurance steps in. Because deposit insurance is not fairly priced, a deposit insurance subsidy arises, distorting banks' investment incentives. Capital requirements reduce the deposit insurance subsidy (a common feature in many models of bank capital regulation following Kareken and Wallace, 1978) but also reduce lending when bank equity is scarce.

We first provide a positive analysis of exogenous policy interventions, focusing on the two most commonly proposed tools, the *green supporting factor* (lower capital requirements for clean loans) and the *brown penalizing factor* (higher capital requirements for dirty loans). Both of these interventions decrease the relative profitability of making a dirty loan (similar to a substitution effect). However, they have opposite marginal effects on credit allocation. Whereas a brown penalizing factor crowds out the bank's marginal loan, a green supporting factor leads to crowding in at the margin (similar to an income effect). One significant implication is that raising capital requirements for dirty loans crowds out clean lending if the marginal loan is clean. Our baseline model with two types separates these two effects particularly cleanly. However, the economic insights carry over to more general settings with many types: The net effect of green tilts to prudential capital requirements on bank funding decisions depends on the relative size income and substitution effects.

Building on our positive analysis of exogenous policy changes, we then characterize how to optimally account for climate-related financial risks under a strictly prudential mandate. The prudential regulator's objective is to maximize the NPV generated by bank-funded firms net of deadweight costs arising from the deposit insurance put. Because the prudential regulator does not care about carbon emissions per se, emissions are reflected in capital requirements only insofar as they correlate with the NPV of the firm's investment and the associated deposit insurance put. Our analysis shows that capital requirements can effectively address climate-related financial risks. This reflects the broader insight that, conceptually, this is no different from managing "traditional" risks. The main difference is one of measurement, given that historical data series contain limited information about the nature of climate-related financial risks.

We illustrate optimal prudential capital requirements in a transition risk scenario in which, due to changes in consumer preferences or environmental regulation, dirty firms become less profitable and riskier relative to a pre-climate risk calibration. The prudential regulator responds to these additional risks by increasing capital requirements for dirty loans, sometimes coupled with a decrease in the capital requirement for clean loans. Notably, when climate-related risks for dirty firms are small, it can be optimal for the

prudential regulator to increase capital requirements for dirty loans even if this crowds out lending to marginal clean firms. In this case, the prudential regulator does not act to reduce lending to dirty firms, but simply finds it optimal to require more capital for these loans in order to reduce their deposit insurance put. This prediction changes in the presence of large climate-related risks. In this case the prudential regulator uses capital requirements to change bank's private ranking of loan types and induce them to fund clean firms first.

We then turn to capital requirements as a tool to address broader carbon externalities. These include direct externalities of carbon emissions on agents in the economy (including future generations) as well as financial risks that are generated by emissions of bank-funded firms but materialize outside of the regulator's perimeter, so that they are not captured by the regulator's prudential mandate (e.g., physical risks that mainly affect firms and banks in other parts of the world). The regulator accounts for such externalities only under a broader "impact" mandate, which adds concern for externalities to the usual prudential objective.

Our analysis reveals that capital requirements are a not an effective tool to address carbon emissions. Even at capital requirements of 100%, banks may find dirty loans financially profitable. If the banking sector is sufficiently well-capitalized, this means that capital requirements cannot prevent the funding of dirty loans. If bank equity is scarce, capital requirements can prevent the funding of dirty loans, but it may be necessary to lower the capital requirements for clean loans below the prudentially optimal level, thereby sacrificing financial stability. If (some) dirty firms have access to alternative sources of financing (e.g., via the bond market) the impact regulator is further constrained by leakage due to substitution to other funding markets. (In contrast, a prudential regulator would welcome substitution to the bond market because it removes risk from the banking sector.)

Carbon taxes (or other policy measures that directly affect the profitability of carbon-intensive activities) do not face the same constraints and are, therefore, better suited to address carbon externalities. However, governments may be reluctant to introduce carbon taxes if the resulting revaluation of legacy assets could trigger a banking crisis. Even worse, anticipating this, banks have no incentive to reduce their carbon exposure, leading to an inefficient regulatory standstill. If government inaction results from such a commitment problem, capital requirements can play an indirect role in reducing carbon emissions. By creating sufficient loss-absorbing capital in the banking sector, they make carbon taxes credible, thereby facilitating government intervention. Therefore, even though our results do not support the use of capital requirements to replace carbon

taxes or other forms of government intervention, they point to one specific channel in which they can facilitate government action by removing stranded asset risk.

Related literature. To the best of our knowledge, this paper is the first to study the positive and normative implications of climate change for bank capital regulation. Our framework builds on the large literature on prudential bank capital regulation.[3] This literature has focused on capital regulation in the presence of distortions introduced by deposit insurance, but has not considered how climate changes affects capital requirements, which is the central focus of our paper. Introducing climate change leads two major departures from this literature. First, climate-related risks (see, e.g., Giglio, Kelly and Stroebel (2021)) become relevant for prudential bank capital regulation insofar as they affect financial risks in the banking sector. Second, climate change may lead to a change in the regulatory objective function to include carbon externalities, in addition to prudential risks in the banking sector. In this respect, our model is related to Thakor (2021), who develops a model of bank capital regulation in which the regulator's objective includes political considerations.

Our analysis of optimal capital regulation is complementary to Dávila and Walther (2022), who develop a general model of optimal second-best regulation, with an application to financial regulation in the presence of environmental externalities. Two recent papers have investigated positive effects of green capital requirements but do not consider optimal (green) capital regulation: Dafermos and Nikolaidi (2021) study green differentiated capital requirements in a dynamic macrofinance model. Thomä and Gibhardt (2019) estimate the effect of green supporting and brown penalizing factors on required bank capital, assuming that the composition of bank balance sheets is unaffected by such a policy change.

While the focus of our paper is on bank capital regulation, Papoutsi, Piazzesi and Schneider (2021) study the environmental impact of central bank asset purchases. Whereas bond purchases affect mainly firms that rely disproportionately on bond financing, bank capital regulation has the strongest effect on bank-dependent firms. Our result that capital requirements have limited ability to deter loans to dirty companies is reinforced if banks are worried that investing in (new) green loans will devalue dirty legacy assets, as pointed out by Degryse, Roukny and Tielens (2022). Jondeau, Mojon and Monnet (2022) propose a liquidity backstop to prevent runs on brown assets that may occur as

[3] This literature includes, among others, Kareken and Wallace (1978), Rochet (1992), Repullo (2004), Pennacchi (2006), Repullo and Suarez (2004), Allen, Carletti and Marquez (2011), Admati, DeMarzo, Hellwig and Pfleiderer (2011), Martinez-Miera and Suarez (2012), Acharya, Mehran and Thakor (2015), Bahaj and Malherbe (2020, 2022), Malherbe (2020), Begenau (forthcoming) and Harris, Opp and Opp (2020).

part of the transition toward a greener economy.

1 Model

We consider a model with two dates ($t = 0, 1$), universal risk-neutrality, and no time discounting. There are three types of agents: a continuum of firms with investment opportunities, a continuum of competitive banks, and a regulator who sets capital requirements.

Firms. Each firm is of infinitesimal size and born to be one of two observable types, $q \in \{C, D\}$, which we will refer to as clean and dirty. The mass of firms is normalized to one, and the population fraction of type q is given by π_q. In Section 5, we discuss the possibility that firms can change their type and become cleaner at a cost, as in Oehmke and Opp (2019).

Firms are born cashless and, for each type, production requires an investment of fixed scale I at $t = 0$. At date $t = 1$, cash flows are realized. We assume that cash flows for each type q, $X_q(s)$, follow a log-normal distribution with expected cash flow $\overline{X}_q$ and asset return volatility σ_q. (We assume log-normality for tractability; our model predictions are independent of the specific distributional assumption.) In our baseline model, we assume equal volatility parameters, $\sigma_q = \sigma$. Cash flows are perfectly correlated within each type and can have arbitrary correlation across types. Both firm types have positive NPV investment opportunities, $\text{NPV}_q := \overline{X}_q - I > 0$, but dirty firms are more profitable than clean firms,

$$\text{NPV}_D > \text{NPV}_C, \tag{1}$$

reflecting the additional costs incurred by clean firms (e.g., carbon capture). While dirty firms are financially more profitable, they produce higher carbon emissions than clean firms, $\phi_D > \phi_C = 0$ (where we normalize carbon emissions of clean firms to zero).

This baseline scenario aims to capture that, at least historically, there has been a tension between profitability and sustainability, for example because of absent or imperfect carbon taxes (see, e.g., Dewatripont and Tirole (2020) and Tirole (2012)). A significant increase in carbon taxes is one important source of transition risks that will differentially affect dirty firms (in the form of lower expected cash flows $\overline{X}_q$ or higher volatility σ_q). We discuss the effects of such transition risks as well as externalities caused by firms' emissions in the context of optimal capital requirements in Section 4.2.

Banks. Firms can raise funds for production by obtaining a loan from a continuum of competitive and ex-ante identical banks (also of mass one). Each bank is endowed with

inside equity $E < I$. Because there is a unit mass of banks, E also corresponds to the aggregate amount of equity in the banking sector.[4] To finance assets A, each bank can raise additional deposit funding D from competitive depositors, resulting in the balance sheet identity

$$A = E + D. \tag{2}$$

Bank capital structure matters because the model features two deviations from the Modigliani-Miller benchmark. First, we assume that outside equity issuance is subject to frictions. For ease of exposition, we assume that the associated issuance cost is prohibitively high, so that bank equity is fixed at E.[5] Second, deposit insurance (or an implicit or explicit bailout guarantee for debtholders) results in an effective subsidy for deposit financing.[6] In our model, deposit insurance is not priced, so that total payouts to bank security holders are increasing in the deposit-to-asset ratio $\frac{D}{A}$. The results would be similar if deposit insurance were priced imperfectly, as in Chan, Greenbaum and Thakor (1992).[7]

Banks maximize their equity value,

$$V = \max_{e,\mathbf{w}} E\left[1 + r_E\left(\mathbf{w},e\right)\right], \tag{3}$$

where we define $e := \frac{E}{A}$ as the bank's equity ratio and $r_E\left(\mathbf{w},e\right)$ as the bank's expected return on equity (ROE), and where $\mathbf{w} = (w_C, w_D)$ denotes the vector of bank loan portfolio weights for clean and dirty loans, respectively. Given that bank equity E is fixed, this objective function boils down to maximizing bank ROE. (Note that in our risk-neutral setting, any ROE exceeding 0 reflects a scarcity rent rather than a risk premium.)

Bank Regulator. The bank regulator sets loan capital requirements $\underline{e}_q$ as a function of the (observable) firm type q.[8] Given a bank's loan portfolio weights w_q, the bank then

[4] The restriction that $E < I$, therefore, rules out the case in which all firms can receive funding even if capital requirements are set to 100% for all loans.

[5] Banks could theoretically pay out part of their equity capital as dividends. However, as we will see below, under optimal regulation it is never optimal for banks to pay dividends. Our results remain qualitatively unchanged if banks can issue additional outside equity at a positive but non-prohibitive marginal cost (see the discussion in Section 5).

[6] For ease of exposition, we simply assume assume the presence of deposit insurance or, alternatively, and implicit bailout guarantee. Deposit insurance arises naturally in banking models with fragility, following Diamond and Dybvig (1983). Dávila and Goldstein (forthcoming) propose a model of optimal deposit insurance. Bianchi (2016) and Chari and Kehoe (2016), among others, develop models of endogenous bailouts.

[7] One may wonder why we assume both a cost of outside equity and a (private) benefit of debt, given that either of these frictions would be sufficient to ensure that banks favor debt financing. The reason is that, in the absence of costly equity issuance, the regulator could simply eliminate bailout distortions by setting capital requirements to 100%.

[8] It is not crucial for our results that firm types are perfectly observable. The main results continue

faces an equity-ratio constraint

$$e \geq e_{\min}(\mathbf{w}) := \sum_q w_q \cdot \underline{e}_q. \tag{4}$$

Capital requirements have two effects. First, higher capital requirements reduce transfers from the deposit insurance fund. When analyzing optimal capital requirements in Section 4, we will follow Farhi and Tirole (2020) by assuming that these transfers are associated with a deadweight cost (e.g., due to a positive shadow cost of public funds). Second, a higher capital requirement for a firm of type q affects banks' loan decisions and, therefore, the mass of funded firms, ω_q.

2 Equilibrium with Exogenous Capital Requirements

We start by characterizing the banking sector equilibrium for exogenously given capital requirements. This analysis will form the basis of our analysis of green tilts to capital requirements in Section 3 and optimal capital requirements in Section 4. The analysis in this subsection draws on Harris et al. (2020), and we therefore present the results in a heuristic fashion. All proofs can be found in Appendix A.

We first characterize optimal decisions by individual banks and then characterize equilibrium lending by the banking sector as a whole.

Result 1 *The regulatory leverage constraint $e^* = e_{\min}(\mathbf{w}^*)$ binds and individual banks find it optimal to specialize in funding either clean or dirty firms.*

Result 1 states that individual banks maximize the amount of deposit funding and choose specialized portfolios.[9] This is optimal because deposit insurance generates a subsidy for deposit funding. Specialization maximizes this subsidy. The value of this deposit insurance put is passed on to bank equityholders via the competitive pricing of deposits.

It is useful to frame the banking sector equilibrium in terms of aggregate bank equity. Bank equity E is the scarce resource in the economy. Therefore, drawing an analogy to demand theory, a firm borrowing from a bank is similar to a consumer, and the relevant consumption good is units of bank equity (i.e., space on the bank's balance

to hold if the regulator observes a noisy signal of firm type (see Section 5).

[9] While this prediction is somewhat extreme, specialization is analytically convenient because it allows us to derive closed-form solutions. Economically, all main insights carry through to the case in which banks' loan portfolios are not specialized.

sheet). Specifically, when a firm of type q demands a loan of size I, this translates into demand for $I\underline{e}_q$ units of bank equity.

Given objective function (3), banks rank borrowers according to the maximum ROE associated with a loan to a borrower, which is determined by the maximum interest rate the borrower is willing to pay. As in standard demand theory, the demand curve is then characterized by "reservation prices," in the form of the maximum return on equity that a borrower can offer to a bank.

Result 2 *At the maximum interest rate that a borrower of type q is willing to pay, the bank equityholders' expected ROE is given by*

$$r_q^{\max}\left(\underline{e}_q\right) = \frac{NPV_q + PUT_q\left(\underline{e}_q\right)}{I\underline{e}_q}, \tag{5}$$

where $PUT_q\left(\underline{e}_q\right)$ denotes the contribution of the loan to the bank's deposit insurance put,

$$PUT_q\left(\underline{e}_q\right) = \mathbb{E}\left[\max\left\{I(1-\underline{e}_q) - X_q(s), 0\right\}\right]. \tag{6}$$

At the borrower's reservation interest rate, all expected surplus generated by the loan accrues to bank equityholders.[10] This surplus consists of the NPV of the firm's project and the value of the deposit insurance put associated with the loan under optimal (maximum) leverage, see Result 1. Given the log-normal cash flow distribution, the value of the deposit insurance put can be readily determined by applying the Black-Scholes formula (this follows Merton (1977), as described in Lemma A.1 in the Appendix).

Because banks behave competitively, they typically cannot extract all surplus from borrowers. Instead, the equilibrium return on bank equity r_E^* is pinned down by the intersection of the aggregate demand for bank equity (from funded loans) and its (fixed) supply. The resulting equilibrium is illustrated in Figure 1. Given two borrower types with different reservation prices, the demand curve is a step function.[11] In the illustrated equilibrium, dirty borrowers (red) are fully funded (they are inframarginal), whereas clean borrowers (green) are only partially funded (they are marginal). Since both types are

[10] If borrowers had access to non-bank financing, say via a competitive bond market, then this outside option would pin down the maximum interest rate the borrower is willing to pay for a bank loan (see the Proof of Result 2 and Section 5 for a discussion). In our baseline model firms are bank-dependent for simplicity. Therefore, the outside option is not to invest at all and, therefore, equal to zero.

[11] The banking sector therefore allocates funds as if banks themselves had access to two linear production technologies (clean and dirty) with capacity constraints, resulting in a production function that resembles a Leontief technology: $F(\omega_q) = X_q(s)\min(\omega_q, \pi_q)$ for each type (see also Donaldson, Piacentino and Thakor (2018)). As we discuss in Section 3.3, our results generalize to settings with more types and/or more general production functions.

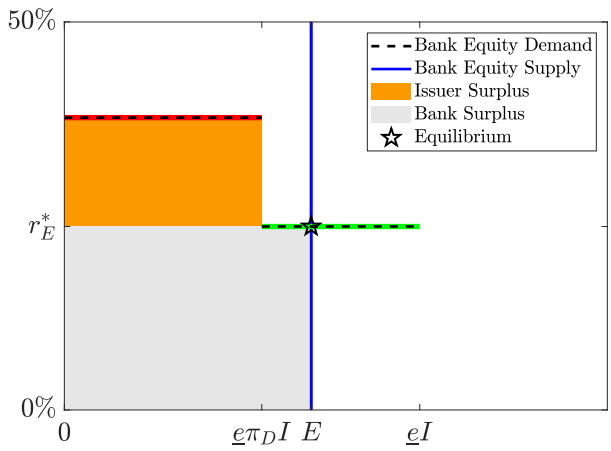

Figure 1. Banking Sector Equilibrium. This figure illustrates the banking sector equilibrium for equal capital requirements of $\underline{e} = 16\%$ for both firm types. The equilibrium ROE is denoted by r_E^*. Dirty firms are depicted in red, clean firms in green.

funded in equilibrium, Result 1 implies that a subset of banks will specialize in funding all dirty firms and the remaining banks will finance exclusively clean firms. The loan rate for the marginal green borrowers is set such that all surplus accrues to banks (i.e., there is no consumer surplus for marginal loans). Inframarginal borrowers, on the other hand, obtain some consumer (or "issuer") surplus, which ensures that banks are indifferent between funding either type. More generally, we obtain

Result 3 *If $E < I \sum_q \pi_q \cdot \underline{e}_q$, bank capital is scarce so that $r_E^* > 0$. All borrowers with $r_q^{max} > r_E^*$ are fully funded by banks. Marginal borrower types, satisfying $r_q^{\max} = r_E^*$, are partially funded. The banking sector's equilibrium ROE satisfies:*

$$r_E\left(\mathbf{w}^*, e_{\min}\left(\mathbf{w}^*\right)\right) = r_E^*. \tag{7}$$

If $E \geq I \sum_q \pi_q \cdot \underline{e}_q$, all types are fully funded and bank capital is not scarce, $r_E^* = 0$.

This result highlights the importance of the marginal borrower type, which pins down r_E^* and, hence, the funding terms and loan allocation to all inframarginal types $r_q^{\max} > r_E^*$. Which borrower type is marginal depends not only on exogenous firm characteristics (such as the firm's NPV, and the capitalization of the banking sector) but also on the regulator's choice of capital requirements.

3 Green Tilts to Capital Requirements

In this section, we investigate the effects of (exogenous) green tilts to capital requirements, as discussed in policy circles. Green tilts can take the form of a reduction in the capital requirement for clean loans (a *green supporting factor*) or an increase in capital requirement for dirty loans (a *brown penalizing factor*.) For expositional clarity, we consider a benchmark policy regime with equal capital requirements for dirty and clean loans ($\underline{e}_C = \underline{e}_D = \underline{e} < 1$) and study the effects of green tilts relative to this benchmark. While policy interventions analyzed in this section are exogenous, the positive analysis of this section will help clarify the economic forces that are at play, and will therefore continue to be relevant when we derive optimal capital requirements in Section 4.

In the benchmark equilibrium with equal capital requirements, loans to dirty firms rank strictly higher in the aggregate demand curve (i.e., $r_D^{\max}(\underline{e}) > r_C^{\max}(\underline{e}) > 0$), because dirty firms are financially more profitable than clean firms, as illustrated in Figure 1. Accordingly, if bank capital is very scarce ($E < \pi_D \underline{e} I$), the marginal borrower type is dirty and no clean loans are funded. For intermediate levels of bank capital ($\pi_D \underline{e} I < E < \underline{e} I$), all dirty firms are funded, and the marginal loan is extended to a clean firm (the case presented in Figure 1). Finally, in the (somewhat less interesting) case of abundant bank capital, $E > \underline{e} I$, both dirty and clean firms are fully funded.

To analyze the effects of green tilts to capital requirements, it is instructive to interpret a change in the capital requirement for one type of loan as a change in the relative prices of bank balance sheet space for clean and dirty loans. Therefore, in analogy to standard demand theory, we can characterize the effects of green tilts in terms of *income* and *substitution effects*. The substitution effect captures that a green tilt reduces the relative price of providing clean loans, irrespective of whether the green tilt consists of a green supporting or a brown penalizing factor. However, the two policies differ with respect to the income effect. Whereas a brown penalizing factor constrains bank balance sheet space, akin to a negative income effect, a green supporting factor frees up balance sheet space.

3.1 Brown Penalizing Factor

To consider the effects of a brown penalizing factor, it is useful to start with a small intervention (i.e., a marginal increase in the capital requirement for dirty loans relative to the benchmark with equal capital requirements). The case of intermediate bank capital $E \in (\pi_D \underline{e} I, \underline{e} I)$, plotted in the left panel of Figure 2, is most interesting. In this case, a marginal increase in the cost of lending to dirty firms leads to crowding out of lending to

clean firms. Relative to the benchmark (dashed red line), a higher capital requirement for dirty loans reduces their attractiveness and results in a downward shift in the dirty borrower's reservation price (from the dashed to the solid red line), without inducing a change in the ranking of types. In addition, because funding the same number of dirty loans now requires more bank equity, the dirty-loan segment of the demand curve lengthens. As a result, less equity is left over to fund clean loans (a rightward shift of the dashed to the solid green line), resulting in a reduction in lending to clean firms.

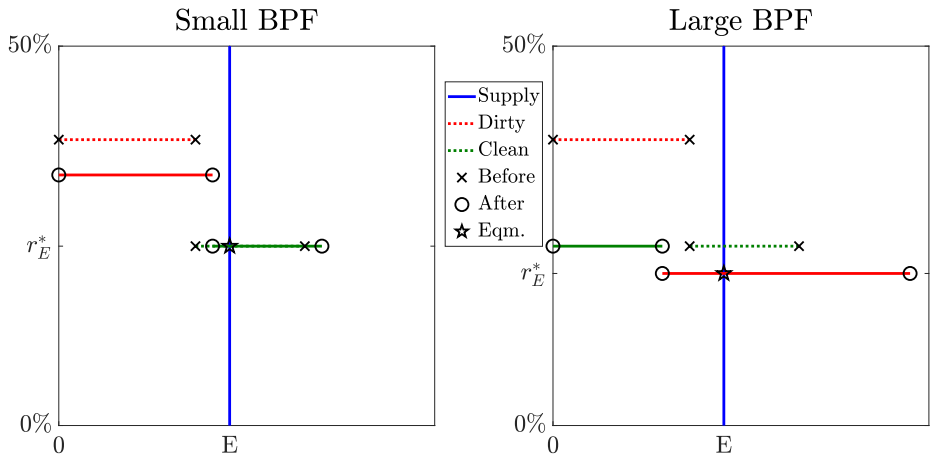

Figure 2. Brown penalizing factor. The figure illustrates the effect of introducing a brown penalizing factor starting from the benchmark of equal capital requirements of $\underline{e} = 16\%$. The left panel illustrates the equilibrium impact of a small brown penalizing factor that leaves the relative ranking of firm types unchanged. The right panel illustrates the equilibrium impact of a large brown penalizing factor that reverses the relative ranking of firm types. Dotted lines and segment endpoints marked $\circ$ denote the benchmark equilibrium. Solid lines and segment endpoints marked $\times$ denote the equilibrium after the introduction of the brown penalizing factor.

If the brown penalizing factor is sufficiently large, the ranking of clean and dirty loans in terms of the borrower reservation price is reversed, so that $r_C^{\max} > r_D^{\max}$ post intervention. This case is illustrated in the right panel of Figure 2. In this case, banks first exhaust all clean lending opportunities before they start funding of dirty firms. Therefore, clean lending increases and dirty lending decreases. This result is driven by both the substitution effect (clean loans get funded first) and the income effect (the lengthening of the dirty-loan segment of the demand curve).

We summarize the effects of the introduction of a brown penalizing factor in Proposition 1. Part 1 of the proposition characterizes the effects of a marginal brown penalizing factor (including the cases of very scarce or abundant bank capital, which we omitted for brevity in the discussion above). Part 2 of the proposition characterizes the effects of a large brown penalizing factor.[12]

[12] To reduce the number of cases in part 2 of the proposition, we assume that the reservation price for

Proposition 1 (Brown Penalizing Factor) *Relative to a benchmark with equal capital requirements for clean and dirty loans:*

1. *The effect of a marginal increase in capital requirements for dirty loans (marginal BPF) depends on the capitalization of the banking sector:*

   (a) *If* $E < \pi_D \underline{e} I$*, a marginal BPF only reduces lending to dirty firms.*

   (b) *If* $E \in (\pi_D \underline{e} I, \underline{e} I)$*, a marginal BPF only reduces lending to clean firms.*

   (c) *If* $E > \underline{e} I$*, a marginal BPF does not affect lending.*

2. *If the increase in capital requirements for dirty firms exceeds a cut-off* $\Delta_{BPF} > 0$*, characterized by* $r_D^{\max}(\underline{e} + \Delta_{BPF}) = r_C^{\max}(\underline{e})$*, lending to clean firms increases whereas lending to dirty firms decreases.*

3.2 Green Supporting Factor

We now turn to the introduction of a green supporting factor. Mirroring the analysis of the brown penalizing factor, we initially consider the introduction of a small green supporting factor.

As before, the intermediate equity case, $E \in (\pi_D \underline{e} I, \underline{e} I)$, is the most interesting. In this case, a small decrease in the capital requirement for clean loans unambiguously increases funding of clean firms, as illustrated in the left panel of Figure 3. While the dirty-loan segment of the demand curve is unchanged, the clean-loan segment shifts upward (albeit without changing the relative ranking of borrower reservation prices) and shortens. The upward shift reflects that clean loans become relatively more attractive in terms of their reservation price, whereas the shortening of the clean-loan segment reflects that each clean loan now requires less capital. As long as the ranking of reservation prices does not change, the equilibrium effect is driven entirely by the income effect (i.e., the shortening of the clean-loan segment). Therefore, some previously unfunded clean firms are now able to obtain funding. The funding of dirty firms is unaffected.

Consider now a green supporting factor that is large enough to reverse the relative ranking of clean and dirty loans, so that $r_C^{\max} > r_D^{\max}$ post intervention. This case is illustrated in the right panel of Figure 3. For clean firms, the income and the substitution effect both push towards strictly more financing. The clean-loan segment of the demand

clean loans exceeds the reservation price for dirty loans when the capital requirement for dirty loans is raised to 100%, i.e., $r_D^{\max}(1) < r_C^{\max}(\underline{e})$. This assumption is satisfied if the expected cash flow of dirty firms NPV_D is sufficiently close to that of clean firms NPV_C.

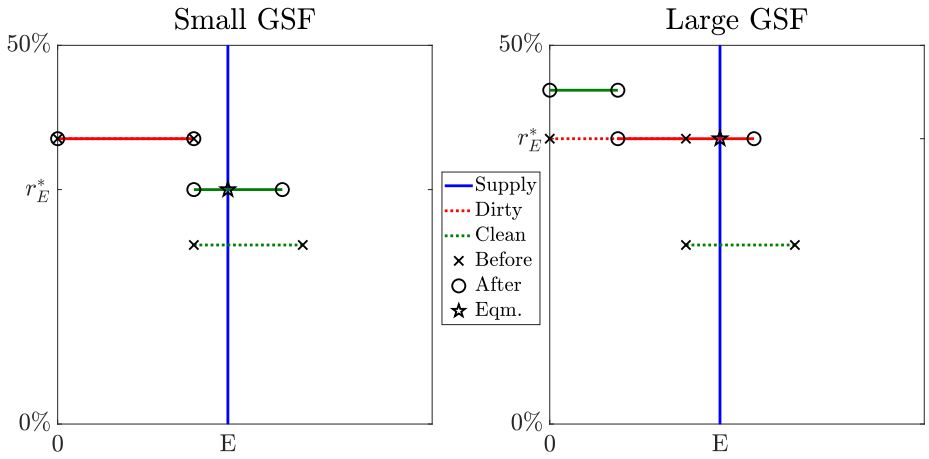

Figure 3. Green supporting factor. The figure illustrates the effect of introducing a green supporting factor starting from the benchmark of equal capital requirements $\underline{e} = 16\%$. The left panel illustrates the equilibrium impact of a small green supporting factor that leaves the relative ranking of firm types unchanged. The right panel illustrates the equilibrium impact of a large green supporting factor that reverses the relative ranking of firm types. Dotted lines and segment endpoints marked $\circ$ denote the benchmark equilibrium. Solid lines and segment endpoints marked $\times$ denote the equilibrium after the introduction of the green supporting factor.

curve shifts upward and shortens, so that after the intervention all clean loans are funded. For dirty loans, the income and substitution effects work in opposite directions. The dirty-loan segment shifts to the right (substitution effect) but this shift is attenuated by the shortening of the clean-loan segment (income effect). After introduction of the large green supporting factor (see right panel), dirty loans take up the balance sheet space that remains after all clean loans have been funded. Dirty loans are marginal. A further reduction in the capital requirement for clean firms would now crowd in dirty loans, implying a non-monotonic effect.

We summarize the effects of a green supporting factor, including the cases of very scarce or abundant bank capital, in Proposition 2.

Proposition 2 (Green Supporting Factor) *Relative to a benchmark with equal capital requirements for clean and dirty loans:*

1. *The effect of a marginal decrease in capital requirements for clean loans (marginal GSF) depends on the capitalization of the banking sector:*

   (a) *If $E \in (\pi_D \underline{e} I, \underline{e} I)$, a marginal GSF increases lending to clean firms.*

   (b) *A marginal GSF has no effect on bank lending otherwise.*

2. *If the decrease in capital requirements for clean firms exceeds a cut-off $\Delta_{GSF} > 0$, characterized by $r_D^{\max}(\underline{e}) = r_C^{\max}(\underline{e} - \Delta_{GSF})$, lending to clean firms increases*

whereas lending to dirty firms decreases, strictly so if not all firms are financed post intervention.

In sum, Propositions 1 and 2 show that, for small interventions, brown penalizing and green supporting factors have quite different effects, driven by opposite income effects. For large interventions, on the other hand, the substitution effect becomes dominant, so that their effects are qualitatively similar.

3.3 Generalizing the Model: Many Firm Types

The above analysis focused on a framework with only two types of homogeneous firms. This baseline setup is helpful in isolating the relevant economic forces in a particularly tractable manner. However, our framework can be adapted to account for any number of types. As in our two-type setup, the banking sector would then rank firm types q according to their reservation price $r_q^{\max}$. The key difference is that income and substitution effects will now typically both be present. The main message remains that changes to capital requirements of infra-marginal borrower types will crowd out lending to the marginal borrower type. For example, if the economy is populated by clean and dirty firms, each with a distribution of productivity levels (effectively generating a decreasing-returns-to-scale production function for clean and dirty firms), a small change to capital requirements will affect a fraction of both clean and dirty borrowers. In our two-type setup, we obtain the extreme (binary) corner cases, in which crowding out either affects only clean or only dirty firms.

Figure 4 illustrates a setting with a finite (but large) number of types, where the mean cash flow of clean and dirty firms $\bar{X}_q$ is drawn from a log-normal distribution, retaining the assumption that dirty firms are more profitable on average. Note that, given random productivity draws, some firm ROE's are negative. These firms are never financed, irrespective of the amount of equity in the banking system.

The upper panel plots the resulting equilibrium for equal capital requirements. In contrast to the baseline model, the demand function is now (approximately) continuous. Because dirty firms are on average more productive, under equal capital requirements they are located mainly on the high-ROE segment of the demand curve. The lower panel illustrates the effect of introducing a brown penalizing factor. Relative to the original equilibrium (retained in black for comparison), the demand curve rotates and lengthens. Moreover, dirty firms look less attractive on average, allowing more clean firms to move into the funded region to the left of the supply (this can be seen from the fraction of dirty firms in each firm decile, displayed on the x-axis). In the illustrated example, the

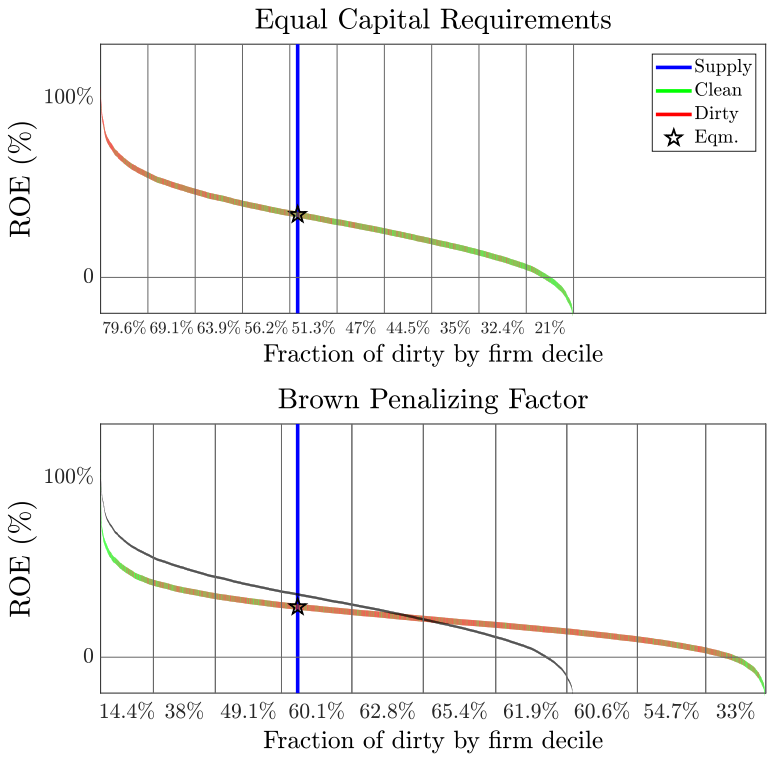

Figure 4. Many types. This figure illustrates an extension of our baseline model, in which the mean cash flow of clean and dirty firms $\bar{X}_q$ is drawn from a log-normal distribution, retaining the assumption that dirty firms are more profitable on average. The top panel illustrates the equilibrium for equal capital requirements of 16% for clean and dirty firms. The bottom panel illustrates the effect of introducing a brown penalizing factor, raising the capital requirement for dirty loans to 29% (this corresponds to the large BPF in Figure 2). In this example, the substitution effect dominates, so that the brown penalizing factor increases credit to clean firms and reduces credit to dirty firms.

substitution effect dominates, so that the brown penalizing factor increases credit to clean firms and reduces credit to dirty firms.

4 Optimal Capital Requirements

Up to now, our analysis has focused on two ad hoc interventions, the brown penalizing and green supporting factors, starting from a benchmark equilibrium with exogenously given symmetric capital requirements. In this section, we turn to optimal capital requirements.

We first characterize optimal prudential capital requirements in the absence of climate-related risks. This benchmark can be interpreted as regulation that is calibrated in a backward-looking fashion using data from a time period in which climate-related risks have not played a significant role. We then analyze how different categories of climate-related risks affect optimal capital requirements under a *prudential mandate*. Finally, we

consider a regulator with a broader *impact mandate* that, in addition to the prudential mandate, accounts for carbon externalities that materialize outside the banking system. The merits of such an impact mandate are being currently discussed controversially in regulatory circles.

4.1 The Principles of Prudential Regulation

The prudential regulator trades off the financial value (or NPV) created by bank lending against the deadweight costs generated by deposit insurance. For simplicity, we assume that the deadweight cost of the deposit insurance put is linear in the size of the fiscal transfer to the banking sector, reflecting a constant marginal cost of public funds λ. The regulator's objective function is therefore

$$\max_{\underline{\mathbf{e}}} \Omega_P = \max_{\underline{\mathbf{e}}} \sum \omega_q \left(\underline{\mathbf{e}}\right) \left[\mathrm{NPV}_q - \lambda \cdot \mathrm{PUT}_q(\underline{e}_q)\right], \tag{8}$$

where the notation $\omega_q\left(\underline{\mathbf{e}}\right)$ and $\mathrm{PUT}_q(\underline{e}_q)$ highlights the dependence of bank funding decisions and the deposit insurance put on the capital requirements for clean and dirty firms, $\underline{\mathbf{e}} = (\underline{e}_C, \underline{e}_D)$.

To characterize optimal prudential capital requirements, it is instructive to rewrite the regulator's objective function as

$$\max_{\underline{\mathbf{e}}} \Omega_P = E \max_{\underline{\mathbf{e}}} \sum \tilde{\omega}_q \left(\underline{\mathbf{e}}\right) \mathrm{PPI}_q(\underline{e}_q), \tag{9}$$

where $\tilde{\omega}_q \in [0,1]$ reflects the fraction of total equity that the banking sector allocates to funding type q, and where $\mathrm{PPI}_q(\underline{e}_q)$ denotes the *prudential profitability index*. In analogy to the banker's maximal ROE given in equation (5), the PPI reflects the surplus created per unit of bank equity as seen from the prudential regulator's perspective,

$$\mathrm{PPI}_q(\underline{e}_q) = \frac{\mathrm{NPV}_q - \lambda \cdot \mathrm{PUT}_q(\underline{e}_q)}{I\underline{e}_q}. \tag{10}$$

Comparing equations (5) and (10), we see that there are two main differences between the regulator's PPI and the bankers' maximal ROE. First, the deposit insurance put enters with opposite sign, reflecting the wedge between prudential preferences and those of the banking sector. Second, while banks take ROEs as given, the regulator internalizes that the PPIs for each type are affected by the chosen capital requirement.

We assume that the cost of public funds is sufficiently high, $\lambda > \max_q \frac{\mathrm{NPV}_q}{\mathrm{PUT}_q(0)}$, which ensures that the PPI is bounded above for each type (see Lemma A.3 in the appendix).

The capital requirement that maximizes the PPI for type q, $\underline{e}_q^{\text{PPI}}$, satisfies the first-order condition

$$\text{PPI}_q(\underline{e}_q) = \lambda \left| \frac{\partial \text{PUT}_q}{\partial \underline{e}_q} \right| / I. \tag{11}$$

The left-hand side captures the marginal benefit of lowering capital requirements. More firms can be financed, resulting in additional prudential surplus. The right-hand side captures the marginal (social) cost of lower capital requirements for all (inframarginal and marginal) firms, in the form of a higher deposit insurance put per unit of investment I.

From the regulator's perspective, a borrower with a higher PPI delivers more "bang for the buck" (prudential value per unit of equity capital) and is therefore preferred. As shown in Proposition 3 below, the PPI plays an important role for the characterization of optimal prudential capital regulation.

Definition 1 *The regulator's preferred type is the one that achieves the highest possible PPI, i.e.,* $\max_q PPI_q(\underline{e}_q^{PPI})$.

In the baseline case, $\sigma_C = \sigma_D = \sigma$ and $\mu_D > \mu_C$, the prudential regulator's preferred type (highest PPI) is the dirty type D, for two reasons. First, dirty loans create larger financial surplus than clean loans, $\text{NPV}_D > \text{NPV}_C$. Second, given equal variance parameters σ, for any capital requirement $\underline{e}$ dirty loans induce a smaller expected transfer from the deposit insurance fund, $\text{PUT}_D(\underline{e}) < \text{PUT}_C(\underline{e})$.

Proposition 3 (Principles of Optimal Prudential Regulation) *Optimal prudential regulation is characterized by the following four principles.*

P1: All bank equity is used to fund loans,

$$\sum_q \omega_q(\underline{\mathbf{e}})\, \underline{e}_q I = E. \tag{12}$$

P2: For sufficiently low levels of bank equity, $E \leq \pi_D \underline{e}_D^{PPI} I$, the regulator induces banks to lend exclusively to its preferred type D.

P3: If firm type q is partially funded (there is at most one such type), its capital requirement maximizes PPI_q,

$$e_q^* = \underline{e}_q^{PPI}. \tag{13}$$

P4: If both firm types are funded, marginal deposit-insurance puts are equalized across types,

$$\frac{\partial PUT_D}{\partial \underline{e}_D} = \frac{\partial PUT_C}{\partial \underline{e}_C}. \tag{14}$$

Principle P1 states that funded loans fully exhaust the equity of the banking sector. This means that, under optimal prudential regulation, banks do not find it optimal to pay out dividends. Intuitively, this principle helps mitigate the deadweight costs arising from the deposit insurance put. Principle P2 says that the first funded type is the regulator's preferred type D. This ranking can always be made incentive compatible for banks by setting a sufficiently high capital requirement for the (unfunded) clean type. Principle P3 states that the optimal capital requirement for the marginal type maximizes the PPI, $e_q^* = \underline{e}_q^{\text{PPI}}$. Finally, Principle P4 links capital requirements of inframarginal borrowers to those of marginal borrowers. Intuitively, capital requirements for inframarginal borrowers only affect the objective function via the deposit insurance put. As a result, optimality condition (14) requires that marginal reductions in the deposit insurance put resulting from higher capital requirements are equalized across funded types.

In Appendix B, we provide a graphical example that illustrates how these principles of prudential capital regulation determine optimal capital requirements for both types and pin down equilibrium funding decisions. Capital requirements are generally increasing in the capitalization of the banking sector E, which can be interpreted as procyclical capital requirements or, equivalently, countercyclical capital buffers. This result follows from the fact that the banking sector's production technology exhibits decreasing returns to scale, which have to be traded off against the social costs of bailouts.

4.2 The Effect of Climate-Related Risks

We now adapt our baseline model to account for the most relevant effects of climate change (physical risks, transition risks, legal risks, etc.). From the regulator's perspective, it is not only important to gauge the effects of climate change but also their cause, given that carbon emissions (the cause) can be endogenous to financial regulation. Figure 5 illustrates the resulting matrix classification of climate-related risks based on cause (row) and effect (column).

The category "*Bank-Bank*" (**BB**) considers the effects of emissions caused by bank-funded firms on risks in the banking sector. For example, physical risks (e.g., floods or droughts) caused by emissions of bank-funded firms reduce profitability $\overline{X}_q$ and/or increase the cash-flow volatility σ_q of other bank-funded firms. The category "*Bank-Other*" (**BO**) captures risks or externalities generated by the emissions of bank-funded

		EFFECT ON	
		Firms funded by banking sector	Other agents
CAUSED BY	Firms funded by banking sector	Bank-Bank (Section 4.2.3)	Bank-Other (Section 4.2.2)
	Other agents	Other-Bank (Section 4.2.1)	Other-Other (✗)

Figure 5. Climate risk categorization for financial regulators

firms, which then affect firms or consumers outside the bank regulator's perimeter. For example, a bank-funded oil refinery might cause environmental damage that harms the inhabitants of a nearby town. The category "*Other-Bank*" (**OB**) includes risks that are caused outside the banking sector but affect bank-funded firms. For example, transition risks resulting from environmental regulation or shifts in consumer tastes could negatively affect dirty firms (reducing $\overline{X}_D$ and/or increasing σ_D), causing stranded asset risk for banks. Finally, the category "*Other-Other*" (**OO**) covers risks that originate outside the banking sector and affect firms or consumers outside the bank regulator's perimeter.

These categories clarify which risks and externalities bank regulators consider when setting capital requirements according to their regulatory mandates. For example, a prudential regulator takes into account climate risks only insofar as they affect the stability of the banking system. These risks include the categories "Bank-Bank" and "Other-Bank." A regulator who, in addition to prudential concerns, has a mandate to reduce externalities caused by carbon emissions of bank-funded firms also considers the category "Bank-Other." (Neither regulator can address "Other-Other" risks, which is why we disregard this category going forward.)

4.2.1 Exogenous climate-related financial risks (Type OB)

We start by analyzing climate-related financial risks that affect bank-funded firms but are caused by others. For example, regulatory transition risk (e.g., future environmental regulation or the introduction of significantly higher carbon taxes) is, to a large extent, exogenous to bank funding decisions. Stroebel and Wurgler (2021) present survey evidence that regulatory risk is considered the top climate risk over the next five years, which corresponds to the horizon most relevant for capital regulation given the average maturity of bank loans. When transition risk materializes, it lowers the cash flows and in-

creases the downside risk of dirty firms relative to clean firms. Technological risks (e.g., technological obsolescence), stakeholder risks (e.g., changes in consumer or employee preferences), and legal risks (e.g., legal exposures related to emissions or pollution) have similar financial ramifications.

In terms of our model, the effect of such risks on optimal prudential capital regulation can be understood via a comparative statics analysis with respect to the parameters of the cash-flow distribution. In particular, given our assumption of a log-normal cash-flow distribution, we capture transition risks as decreases in the expected cash flow of dirty firms $\overline{X}_D$ and/or increases in their volatility σ_D. There is a debate about the magnitude of these risks and over which horizon they realize. Following the structure of Section 3, we initially consider small changes, now relative to the prudential optimum.

Proposition 4 (Optimal Marginal Policy Adjustments: OB risks) *A marginal increase in the cash-flow volatility of dirty firms σ_D or a marginal reduction in their expected cash flow $\overline{X}_D$*

1. *increases $\underline{e}_D^*$;*
2. *has no effect on $\underline{e}_C^*$ when clean is marginal (Region 3) and decreases $\underline{e}_C^*$ when both types are fully funded (Region 4).*

Intuitively, higher cash-flow volatility increases the put value associated with the dirty type and, hence, raises the marginal prudential cost of funding dirty projects. For reductions in $\overline{X}_D$, this effect on the put value is reinforced by a reduction in NPV. In both cases, the prudential regulator responds by increasing capital requirements for dirty loans. The increase in optimal capital requirements in response to moderate OB risks is illustrated in the left panel of Figure 6 (comparing the solid and dotted red lines). While Figure 6 illustrates OB risks as a reduction in the profitability of dirty firms, an increase in volatility would have virtually the same effect.

Part 2 of Proposition 4 investigates the spillover effects on clean firms originating from climate-related risks that affect only dirty firms. When clean firms are the marginal type (Region 3), capital requirements for clean firms are optimally set to maximize the PPI, $\underline{e}_C^* = \underline{e}_C^{PPI}$, so that optimal prudential capital requirements for clean firms remain unaffected. However, in the region in which both types are fully funded (Region 4), the optimal balancing of marginal puts (Principle P4) requires that capital requirements for clean firms are adjusted downward (comparing the solid and dotted green lines in the left panel of Figure 6).

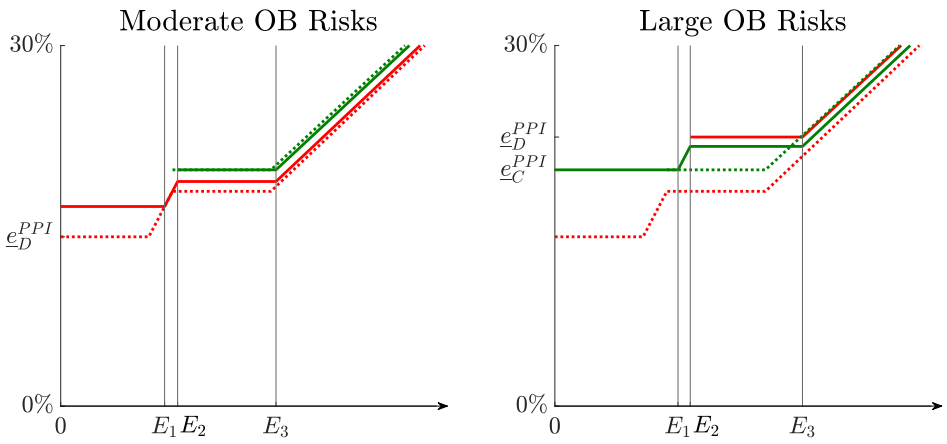

Figure 6. Effects of OB climate risks on optimal prudential capital regulation. The figure plots the effect of other-financial (OB) risks on optimal prudential capital requirements for clean and dirty types. In this example OB risks take the form of a reduction in the expected profitability of dirty firms $\overline{X}_D$, for example, due to transition risk. The left panel illustrates the effect of moderate OB risks, which do not affect the regulator's ranking of firm types. The right panels illustrates the effect of large OB risks that reverse the regulator's ranking of firm types. Solid lines depict optimal capital requirements that take into account OB risks. Dashed lines depict prudential capital requirements that do not take into account OB risks. Note that changes in capital requirements due to OB risks shift the region boundaries relative to Figure B.1.

In sum, the optimal prudential response to moderate climate-related risks of type OB can be implemented with a brown penalizing factor for dirty firms (relative to the prudential baseline without climate risk) and, in some instances, a green supporting factor. As before, the exact calibration of these adjustments depends on the capitalization of the banking sector, as characterized by Proposition 3. This optimal regulatory response has the following allocational consequences.

Corollary 1 (Real Effects of Optimal Marginal Policy Adjustments: OB risks) *The optimal policy response to a marginal increase in the cash-flow volatility of the dirty firms σ_D and/or a marginal reduction in their expected cash flow $\overline{X}_D$*

1. *crowds out lending to dirty firms if bank equity is low, $E < E_1$;*
2. *crowds out lending to clean firms if bank equity is intermediate $E \in (E_2, E_3)$.*

The second part of the corollary states that, perhaps surprisingly, moderate transition risks can make it optimal for a prudential regulator to sacrifice lending to clean firms. To see why this is the case, recall that a prudential regulator cares about climate-related risks only through their effect on firm cash flows and, in turn, financial stability in the banking sector. As long as these effects are moderate in magnitude, the prudential

regulator continues to prefer the dirty type and therefore finds it optimal, at the margin, to tolerate the crowding out of clean lending on financial stability grounds.

This conclusion changes for sufficiently large changes in the relative prospects of clean and dirty types. Such large OB risks result in a more drastic change in optimal prudential policy, even under a purely prudential regulatory objective.

Proposition 5 (Policy Adjustments in Response to Large OB risks) *The regulator's preferred type switches from dirty to clean if*

1. *the expected cash flow of dirty firms* $\overline{X}_D$ *decreases sufficiently;*
2. *the return volatility of dirty firms* σ_D *increases sufficiently and* $\frac{NPV_D}{I} < PPI_C(\underline{e}_C^{PPI})$.

When the regulator's preferred type switches from dirty to clean, funding for clean firms increases and funding for dirty firms decreases.

Intuitively, once climate-related effects on relative cash flows are sufficiently large, clean firms deliver the highest PPI (see Definition 1). This change in the regulator's preferred type implies that prudential capital requirements are optimally set in to reverse banks' ranking of types accordingly. This is illustrated in the right panel of Figure 6. The optimal prudential capital requirements in the presence of large OB risks ensure that clean firms are funded first (up to the cutoff E_2), and dirty firms receive funding only after clean lending opportunities have been exhausted.

In summary, the presence of OB risks that lower the prospects of dirty firms relative to clean firms rationalizes the use of the ad hoc policy tools we analyzed in Section 3. In particular, brown penalizing and green supporting factors become part of the prudential toolkit, and their magnitudes are directly linked to the magnitude of the effect of climate risk on relative cash flow prospects of clean and dirty firms. Whether these optimal adjustments crowd out clean or dirty lending depends on the severity of these risks (comparing Corollary 1 with Proposition 5).

4.2.2 Externalities on other agents (Type BO)

We now consider externalities on other agents. These include direct externalities caused by carbon emissions (e.g., carbon emissions directly affect the utility of agents in the economy) as well as financial risks that are generated by emissions of bank-funded firms but materialize outside of the regulator's perimeter, so that they are not captured by the regulator's prudential mandate (e.g., physical risks that mainly affect firms and banks in other parts of the world). Because the prudential regulator's objective function (8) does not account for externalities on others, we readily obtain

Observation 1 *Optimal prudential capital requirements are unaffected by BO risks.*

Combined with the findings of the previous section, this observation implies that, even in the presence of substantial externalities on others (i.e., significant BO risks), a prudential regulator finds it optimal to tolerate the crowding out of clean lending when climate-related risks affecting the domestic banking sector (OB risks) are moderate (see Corollary 1).

As part of the global policy effort to reduce carbon emissions and the associated externalities, there have been calls that bank capital regulation should go beyond traditional prudential goals and follow a broader "impact"mandate that directly targets carbon externalities (e.g., Dombrovskis, 2017). Assuming, for simplicity, that carbon externalities are linear in emissions, the impact regulator's objective function is then given by

$$\max_{\underline{\mathbf{e}}} \Omega_G = \max_{\underline{\mathbf{e}}} \sum \omega_q \left(\underline{\mathbf{e}}\right) \left[\mathrm{NPV}_q - \phi_q - \lambda \cdot \mathrm{PUT}_q(\underline{e}_q)\right]. \tag{15}$$

In analogy to the PPI, we can then define a social profitability index (SPI), which captures the impact regulator's "bang for buck" including the social costs generated by carbon externalities ϕ_q,

$$\mathrm{SPI}(\underline{e}_q) = \frac{\mathrm{NPV}_q - \phi_q - \lambda \cdot \mathrm{PUT}_q(\underline{e}_q)}{I\underline{e}_q}. \tag{16}$$

The impact regulator's preferred type achieves the highest SPI (rather than PPI). Moderate carbon externalities ϕ_D do not lead to a change in the regulator's preferred type relative to the prudential regulator. In this case, the characterization of optimal capital requirements is similar to the prudential regulator's case under moderate transition risk (replacing $\mathrm{PPI}(\underline{e}_q)$ with $\mathrm{SPI}(\underline{e}_q)$). For the remainder of this section, we will therefore focus on the more interesting (and novel) case in which including carbon externalities leads to a reversal in the regulator's ranking. For concreteness, we focus on the case in which externalities are so large that the SPI becomes negative for dirty firms:

Assumption 1 $\phi_D > NPV_D$.

Under this assumption, carbon emissions are so significant that funding dirty firms reduces social surplus even when capital requirements for dirty loans are set to 100% (i.e., $\mathrm{SPI}_D(1) = \frac{\mathrm{NPV}_D - \phi_D}{I} < 0$). Yet, even at a 100% capital requirement banks find it privately profitable to finance dirty loans because $r_D^{\max}(1) = \frac{\mathrm{NPV}_D}{I} > 0$.

Observation 2 *If $NPV_D > 0$, financing dirty firms is profitable for banks even under maximum capital requirements of 100%.*

This observation illustrates the limitations of capital requirements when it comes to addressing general carbon externalities. Distortions generated by the deposit insurance put can always be dealt with using capital requirements given that the deposit insurance put disappears when the capital requirement is set to 100%.[13] This is not the case in the presence of externalities because, even at capital requirements of 100%, dirty firms with negative social value can attract funding given that

$$r_D^{\max}(1) = \frac{\text{NPV}_D}{I} > 0 > \frac{\text{NPV}_D - \phi_D}{I} = \text{SPI}_D(1). \tag{17}$$

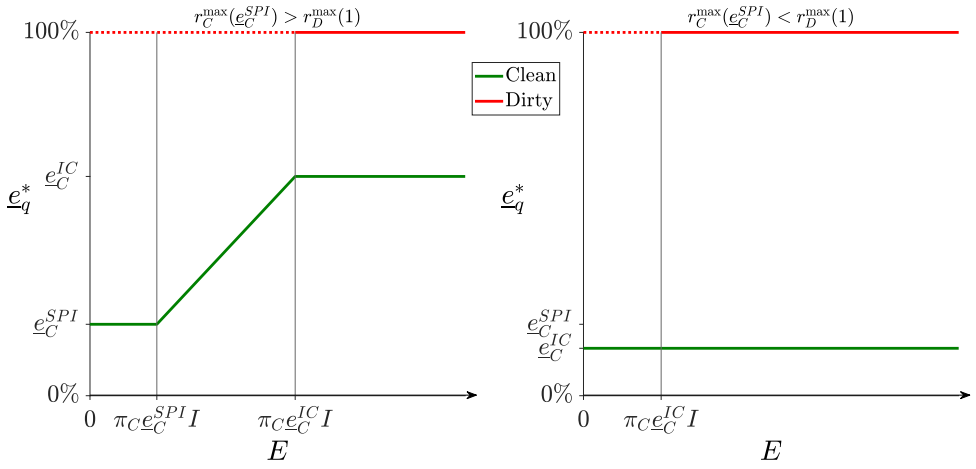

Figure 7. Capital regulation under an impact mandate. This figure plots optimal capital requirements under the green regulator's objective function (15). The left panel illustrates the case, in which banks' IC constraint is not binding. Hence, it is initially possible to set $\underline{e}_C^* = \underline{e}_C^{SPI}$ and to incentivize banks to lend to clean firms by setting capital requirements for dirty firms to 100%. In the right panel, setting $\underline{e}_C^* = \underline{e}_C^{SPI}$ would violate banks' IC constraint and they would therefore lend to dirty firms first. Therefore, the green regulator is forced to lower the capital requirement for clean loans to $\underline{e}_C^* = \underline{e}_C^{IC} < \underline{e}_C^{SPI}$.

Because dirty projects are profitable even at a capital requirement of 100%, the impact regulator's optimal policy is constrained. To see this, consider first the case $r_C^{\max}\left(\underline{e}_C^{SPI}\right) > r_D^{\max}(1)$, as illustrated in the left panel of Figure 7. In this case, the regulator can set the capital requirement for clean loans to maximize SPI while also ensuring that clean loans are funded first (by setting the capital requirement for dirty loans to 100%). For low levels of aggregate bank equity ($E \leq \pi_C \underline{e}_C^{SPI} I$), only clean firms are funded with the clean capital requirement optimally set to $\underline{e}_C^{SPI}$. Once all clean firms have been funded ($\pi_C \underline{e}_C^{SPI} I < E \leq \pi_C \underline{e}_C^{IC} I$), the regulator raises the capital requirements for clean loans

[13] If a firm generates negative prudential value for all levels of $\underline{e}_q$ (i.e., even at a capital requirement of 100%), it must be that $\text{NPV}_q < 0$, since $\text{PPI}_q(1) = \text{NPV}_q / I$. Therefore, the prudential regulator can deter banks from funding this firm by imposing a sufficiently high capital requirement.

to avoid that additional equity is used to fund dirty firms and to lower the deposit-insurance put for clean loans. This increase of clean capital requirement is optimal up to the point at which banks are indifferent between funding clean and dirty loans, $r_C^{\max}\left(\underline{e}_C^{IC}\right) = r_D^{\max}(1)$. If the regulator raised capital requirements for clean loans beyond $\underline{e}_C^{IC}$, banks would prefer to fund dirty rather than clean funding, which explains why capital requirements for clean loans are capped at $\underline{e}_C^{IC}$. As a result, once aggregate bank equity exceeds the amount required to finance all clean firms at a capital requirement of $\underline{e}_C^{IC}$ ($E > \pi_C \underline{e}_C^{IC} I$), banks start to finance dirty firms at $e_D^* = 1$ (as indicated by the solid line for e_D^*). Therefore, irrespective of the size of the associated externalities, capital requirements cannot deter the funding of dirty loans with positive NPV if the banking sector is sufficiently well capitalized.[14]

In the second case, $r_C^{\max}\left(\underline{e}_C^{SPI}\right) < r_D^{\max}(1)$, the regulator cannot set the capital requirement for cleans loans to the level that maximizes the SPI because banks would then prefer to fund dirty loans. For low levels of bank equity ($E \leq \pi_C \underline{e}_C^{SPI} I$), the regulator is therefore forced to lower capital requirements below $\underline{e}_C^{SPI}$ in order to ensure that $r_C^{\max}\left(\underline{e}_C^{IC}\right) = r_D^{\max}(1)$. In this case, it is not possible to induce banks to make clean loans by raising only the capital requirement for dirty loans. The regulator needs to also subsidize clean loans by lowering their capital requirements below the level that would maximize the SPI, up to the point where banks are willing to fund clean loans. This implies that, in order to prevent the funding of dirty loans, the impact regulator has to make a sacrifice with respect to the prudential objective. Moreover (and as in the first case), once aggregate bank equity exceeds the amount required to finance all clean firms at a capital requirement of $\underline{e}_C^{IC}$, $E > \pi_C \underline{e}_C^{IC} I$, capital requirements cannot prevent the funding of dirty firms.

Proposition 6 (The Limits of Green Capital Requirements) *The regulator's ability to use capital requirements to reduce dirty lending is limited.*

1. *If the banking sector is sufficiently well capitalized, $E > \pi_C \underline{e}_C^{IC} I$, capital requirements cannot prevent the funding of dirty loans.*

2. *If bank equity capital is limited, $E \leq \pi_C \underline{e}_C^{IC} I$, capital requirements can prevent the funding of dirty loans. However, if dirty loans are sufficiently profitable, $r_C^{\max}\left(\underline{e}_C^{SPI}\right) < r_D^{\max}(1)$, the regulator has to reduce the capital requirement for clean loans below the prudentially optimal level, $\underline{e}_C^{IC} < \underline{e}_C^{SPI}$, thereby sacrificing financial stability.*

[14] For very high levels of equity, e.g., $E = I$, (outside the plotted region in Figure 7), it is again optimal to equalize marginal puts under full funding for both types, as in Region 4 of Figure B.1 in the appendix.

Our analysis suggests that capital requirements are not an effective tool to address carbon externalities: Even at maximal requirements, banks may find dirty loans financially profitable. This wedge between welfare and bankers' profit motive may require the impact regulator to make sacrifices with respect to financial stability in order to promote clean lending and deter dirty lending. (We contrast capital requirements and carbon taxes in Section 5.)

4.2.3 Endogenous climate-related financial risks (Type BB)

If carbon emissions by bank-funded firms feed back into the cash-flow distributions of bank-funded firms, then these externalities are accounted for by the prudential regulator. For example, physical risks (floods and droughts) caused by the emissions of bank-funded firms can impose negative production externalities in the form of lower cash flows and higher volatility for other (clean and dirty) firms. Competitive banks take these externalities as given and fail to account for their effect on endogenous climate risks. In contrast, the prudential regulator has an incentive to account for the tragedy of the commons by imposing additional capital requirements for dirty firms.

5 Implications

In this section, we discuss how our parsimonious model can be adjusted to make predictions about various empirically relevant extensions.

Non-bank financing. In our model, all firms are bank-dependent. If instead firms had access to competitive public markets (or another alternative source of financing), the formal analysis would be very similar, except that this outside financing option would reduce the borrowers' reservation interest rate. Hence, banks' maximal ROE becomes $r_q^{\max}\left(\underline{e}_q\right) = \frac{\mathrm{PUT}_q\left(\underline{e}_q\right)}{I_{\underline{e}_q}}$ (for details, see the proof of Result 2). Intuitively, the only comparative advantage for banks now stems from government subsidies as reflected in the deposit insurance put.

Note that the assumption of bank dependence gives capital requirements the best shot at addressing externalities: As long as capital requirements can ensure that banks do not fund dirty firms, emissions can be prevented. If (some) dirty firms have access to alternative sources of financing, the impact regulator is constrained by leakage due to substitution to other funding markets. In the language of our risk classification (see Figure 5), the emissions generated by these firms are then no longer "caused" by bank funding

since they would materialize anyway. Whether substitution to non-bank financing is a concern for the regulator depends on the regulatory mandate:

Observation 3 *The impact regulator aims to reduce carbon emissions and therefore would like to prevent substitution to non-bank funding. The prudential regulator welcomes substitution because it removes risk from the banking sector.*

Bank capital scarcity and the cost of raising equity. In our baseline model, bank equity capital is scarce if equity E is sufficiently low. Our qualitative results are unchanged if banks can raise additional equity at a positive marginal cost. However, if the marginal cost of raising additional bank equity is zero (when seen from a social perspective), as argued by Admati et al. (2011), then bank capital is never scarce. In this case, which in our model corresponds to setting E to a sufficiently large value, prudential regulation is no longer subject to a trade-off. Sufficiently high capital requirements (formally $e_q = 1$) eliminate the social cost of bailouts without adverse effects on socially valuable lending.

Interestingly, even though the prudential regulator's trade-off disappears when banks can raise equity at zero cost, abundant bank equity eliminates the impact regulator's ability to use capital requirements as a tool to reduce emissions. When bank capital is abundant, a profitable firm can receive bank funding even when externalities are so large that social value is negative, $\phi_D > NPV_D > 0$.

In sum, capital requirements are an effective tool to reduce carbon externalities only if firms do not have alternative (non-bank) funding sources and if bank capital is sufficiently scarce. If either of these conditions is not satisfied, the impact regulator is powerless to affect emissions.

Carbon taxes and capital requirements In our baseline model, we showed how increases in carbon taxes feed back into optimal capital requirements. A prudential regulator views carbon taxes as a source of transition risk. Propositions 4 and 5 show that optimal prudential capital regulation accounts for this risk by raising capital requirements for carbon-intensive loans.

Proposition 6 implies that carbon taxes are more effective than capital requirements when it comes to reducing externalities. In contrast to capital requirements, sufficiently high carbon taxes can always ensure that investing in firms with negative SPI is unprofitable. Moreover, because carbon taxes are independent of the lender's identity (e.g., banks or public markets), carbon taxes reduce the profitability of lending to dirty firms,

both for banks and non-banks. Therefore, the leakage concern discussed in the previous section is moot.

As a result, our baseline analysis suggests the following pecking order. First, governments should set carbon taxes to address carbon externalities. Then, in a second step, bank regulators deal with the resulting transition risks via capital regulation. However, this prescription presumes that there are no significant impediments to efficient environmental regulation.

One such impediment arises from the inability of governments to commit to future policies. Higher carbon taxes will likely lead to a significant reduction in the value of legacy loans to carbon-intensive firms. Unless the banking sector has a sufficiently large equity cushion to absorb such losses, governments may be reluctant to impose stricter environmental regulations to avoid a banking crisis. In this situation, capital requirements can help facilitate carbon taxes or stricter environmental regulation.

To capture this idea formally, consider a repeated game with the following sequence of events within each period.

- At date 0, the banking regulator sets capital requirements and banks choose their leverage and loan portfolio.
- At date 0.5, the government sets carbon taxes.
- At date 1, firm cash flows are realized and banks can default.

The sequential nature of this game aims to capture the idea that banks are exposed to dirty legacy assets when the government decides on carbon taxes or environmental regulation. At date 0.5, an increase in carbon taxes has a negative short-run and positive long-run effect on welfare. Since banks' lending and capital structure decisions are sunk at date 0.5, the short-run effect of higher carbon taxes is negative because they render dirty firms less profitable and, in turn, increase bank default risk at date 1. The long run welfare effect (in subsequent periods) is positive, because carbon emissions are lower going forward. The trade-off between these short-run costs and long-run benefits determines the optimal environmental policy.[15]

Higher capital requirements for carbon-intensive loans increase the equity cushion against the downside risk arising from higher carbon taxes. They also (weakly) reduce

[15] In line with this prediction, Döttling and Rola-Janicka (2022) develop a model of Pigouvian taxation in the presence of financial constraints and show that welfare can be improved when emission taxes are complemented by leverage limits for firms (there are no banks in their model). Biais and Landier (2022) highlight a complementarity between emissions caps set by the government and (private) green investment.

the amount of carbon-intensive loans on bank balance sheets. Both of these effects reduce the marginal social cost of increasing carbon taxes, making it credible for the government to follow through with higher carbon taxes or more stringent regulation. In sum, if governments are subject to a commitment problem, capital regulation can play an indirect role in reducing carbon emissions by facilitating government action.

Imperfect observability of firm types. For expositional clarity, we assumed that the bank regulator can perfectly observe both the riskiness and emissions of a firm. If the regulator only observed a noisy signal of firm quality, the main results would be qualitatively similar. There are, however, some additional results regarding potential unintended consequences. If, for example, clean firms consisted both of risky clean firms and safe clean firms, a blunt green supporting factor for all clean firms would disproportionately benefit risky clean firms, which would benefit from a larger increase in the value of the deposit insurance put. This could incentivize banks to take on excessive "green risks" (akin to subprime structured securities that were subject to lenient capital charges in the run-up to the Great Financial Crisis.)

Firms' choice of production technology. For ease of exposition, we assumed that firm types are exogenously given, which should be interpreted as firms operating either in a clean or a dirty sector. In this baseline model, green tilts to capital regulation affect emissions via the banking sector's allocation of funding across sectors. If, in addition, firms within a given sector had access to a costly pollution-reducing technology, as in Oehmke and Opp (2019), they may have an incentive to invest in these technologies if capital requirements reward such investments. The incentives to become clean would depend on how much doing so increases in the maximum ROE firms can offer to banks.

6 Conclusion

How should climate change and associated climate risks be reflected in bank capital regulation? This paper has developed a flexible model of capital requirements to investigate both positive and normative aspects of this question.

Our positive results highlight that increases in capital requirements for dirty loans can reduce clean lending. Conversely, decreases in capital requirements for clean loans can crowd in dirty lending. This result obtains because changes in capital requirements affect credit allocation only via the marginal loan. Our model characterizes the conditions under which the marginal loan is clean or dirty.

From a normative perspective, our analysis shows that capital requirements can be an effective tool to deal with prudential risks arising from climate change. However, addressing climate-related financial risks via capital requirements is not equivalent to reducing emissions. For example, it can be optimal for a prudential regulator to increase capital requirements on loans affected by climate-related financial risk even if this crowds out clean lending. The insight that capital requirements can effectively deal with climate-related financial risks reflects that, conceptually, doing so is no different from managing "traditional" risks. However, in contrast to traditional risks, financial risks caused by climate change pose novel measurement challenges because historical data series contain limited information about these risks.

In contrast, using capital requirements to discourage the funding of carbon-intensive activities is less promising. First, as long as activities with high carbon emissions remain profitable, removing loans that fund these activities from the banking sector may either be impossible altogether or may require lowering capital requirements on loans with small carbon footprints below the prudentially optimal level, thereby sacrificing financial stability. Second, even if capital regulation can successfully remove dirty loans from the banking system, high-emitting activities will likely attract funding elsewhere as long as they offer a positive return to investors. Therefore, interventions that directly reduce the profitability of carbon-intensive investments (e.g., a carbon tax) are more effective tools to reduce carbon emissions. In this context, capital requirements can play an indirect role: By ensuring sufficient loss-absorbing capital in the banking sector, they can help facilitate carbon taxes or stricter environmental regulation, which governments may otherwise be reluctant to introduce because the resulting revaluation of bank assets and the associated stranded asset risk could trigger a banking crisis.

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A Proofs

Proof of Result 1: Let $y_q \geq 0$ denote the interest rate that a borrower of type q promises to pay on the loan of size I. (This promised yield will be endogenous in equilibrium, see Results 2 and 3). Then, if a bank lends only to borrowers of type q (i.e., $w_q = 1$) and chooses a feasible equity ratio $e \geq \underline{e}_q$, its expected return on equity can be written as:

$$r_E = \frac{\mathbb{E}\left[\max\left\{\min\left\{I\left(1+y_q\right), X_q(s)\right\} - \left(1-e\right)I, 0\right\}\right] - eI}{eI} \tag{A.1}$$

$$= \frac{\mathbb{E}\left[\max\left\{\min\left\{Iy_q, X_q(s) - I\right\}, -eI\right\}\right]}{eI}. \tag{A.2}$$

Equation (A.1) reflects that the bank receives (from each borrower of type q) the minimum of the promised loan repayment, $I\left(1+y_q\right)$, and borrower's actual cash flows, $X_q(s)$. Given an equity ratio of e, the amount of debt financing (per borrower) is $\left(1-e\right)I$. Since depositors require zero interest on their deposits (due to bailouts/deposit insurance), the bank needs to repay depositors a total of $\left(1-e\right)I$. Since bank shareholders are protected by limited liability, their gross-payoff is bounded below by zero. The numerator, therefore, reflects the expected payoff for bank shareholders net of their co-investment eI. Dividing by the co-investment yields the bank's expected return on equity. We can now decompose the numerator to write (A.2) as

$$r_E = \frac{\mathbb{E}\left[\min\left\{Iy_q, X_q(s) - I\right\}\right] + \mathbb{E}\left[\max\left\{-eI - \min\left\{Iy_q, X_q(s) - I\right\}, 0\right\}\right]}{eI}, \tag{A.3}$$

which follows from $\max\left\{a, b\right\} = a + \max\left\{b - a, 0\right\}$, setting $a = \min\left\{Iy_q, X_q(s) - I\right\}$ and $b = -eI$. Here, $\mathbb{E}\left[\max\left\{-eI - \min\left\{Iy_q, X_q(s) - I\right\}, 0\right\}\right]$ can be interpreted put value arising from a loan to a firm of type q. This put value can be further simplified, since $\max\left\{-eI - \min\left\{Iy_q, X_q(s) - I\right\}, 0\right\} = 0$ whenever the borrower can repay (i.e., for all states s for which $Iy_q > X_q(s) - I$). We thus obtain

$$r_E = \frac{\mathbb{E}\left[\min\left\{Iy_q, X_q(s) - I\right\}\right] + \mathbb{E}\left[\max\left\{I\left(1-e\right) - X_q(s), 0\right\}\right]}{eI}. \tag{A.4}$$

Equation (A.4) shows that the bank's ROE is strictly decreasing in e, so that the bank optimally chooses the minimum equity co-financing $e = \underline{e}_q$. Moreover, mixing two borrower types is strictly dominated because diversification lowers the bank's put value. This reflects the standard result that the option on a portfolio is has (weakly) lower value than the corresponding portfolio of options. ■

Proof of Result 2: Let $y_q^{\max}$ denote the maximum interest rate that a borrower is willing to pay. The maximum ROE from lending to a borrower of type q is achieved by lending with maximum leverage, $e = \underline{e}_q$, at rate $y_q^{\max}$. Equation (A.4) then becomes

$$r_q^{\max}\left(\underline{e}_q\right) = \frac{\mathbb{E}\left[\min\left\{Iy_q^{\max}, X_q(s) - I\right\}\right] + \text{PUT}_q\left(\underline{e}_q\right)}{\underline{e}_q I}, \tag{A.5}$$

where $\text{PUT}_q\left(\underline{e}_q\right) := \mathbb{E}\left[\max\left\{I(1-\underline{e}_q) - X_q(s), 0\right\}\right]$. Equation (A.5) covers both the case in which the firm type is bank bank-dependent (as in our baseline model) and the case in which the firm has access to an outside option (as in Section 5).

Case 1: If the firm is bank-dependent (and, thus, lacks an outside financing option) it is willing to pledge the entire NPV to the bank. (Given that the log-normal cash-flow distribution is unbounded above, this corresponds to $y_q^{\max} = \infty$.) In this case, $\mathbb{E}\left[\min\left\{Iy_q^{\max}, X_q(s) - I\right\}\right] = \mathbb{E}\left[X_q(s) - I\right] = \text{NPV}_q$. Then (A.5) simplifies to (5).

Case 2: If the firm has access to a competitive outside option, the reservation interest rate $y_q^{\max}$ equals the interest rate on the outside option. The value of $y_q^{\max}$ must be such that a competitive outside investor just breaks even on the investment,

$$\mathbb{E}\left[\min\left\{I\left(1+y_q^{\max}\right), X_q(s)\right\}\right] = I, \tag{A.6}$$

which implies that $\mathbb{E}\left[\min\left\{Iy_q^{\max}, X_q(s) - I\right\}\right] = 0$. Therefore, the maximal ROE for bank equityholders (A.5) becomes:

$$r_q^{\max}\left(\underline{e}_q\right) = \frac{\text{PUT}_q\left(\underline{e}_q\right)}{\underline{e}_q I}. \tag{A.7}$$

This expression reflects that the only comparative advantage of banks relative to competitive outside investors results from access to deposit insurance. ■

Proof of Result 3: Given that equity is the (potentially) scarce resource, the banking sector prioritizes borrowers according to the maximum expected ROE (which act akin to reservation prices). We need to distinguish two cases.

Scarce equity: If not all firms can be financed given equity E, then the marginal borrower type q_M pays the maximum interest rate $y_{q_M}^{\max}$ on her loan. A fraction of marginal firms with $r_{q_M}^{\max}\left(\underline{e}_{q_M}\right) = r_E^*$ is rationed. Even though banks are competitive, banks earn a scarcity rent of $r_E^* = r_{q_M}^{\max}\left(\underline{e}_{q_M}\right) > 0$. All borrower types with $r_q^{\max}\left(\underline{e}_q\right) > r_E^*$ are inframarginal and are fully financed. The interest rate on their loan $y_q < y_q^{\max}$ is set below their reservation interest rate, which ensures that banks also earn a ROE of r_E^* on loans to inframarginal borrower types (who, thus, obtain some borrower surplus from their projects).

Non-scarce equity: In this case, banks finance all firms that can offer a positive ROE, $r_{q^*}^{\max}\left(\underline{e}_{q^*}\right) > 0$. Since banks are competitive and equity is not scarce, loan interest rates are such that banks earn a ROE of $r_E^* = 0$ on all loans. All surplus (including the put value) is passed on to borrowers. ■

Proof of Proposition 1: Given equal capital requirements, we obtain that $r_D^{\max}(\underline{e}) > r_C^{\max}(\underline{e})$. This follows from our (baseline) assumption that the cash flow distribution of dirty firms first-order stochastically dominates the cash flow distribution of clean firms. It is easiest to see this using Equation (A.1) and setting $e = \underline{e}$ (see Result 1).

1.a If $E < \pi_D \underline{e} I$, the dirty firm type is marginal. (The threshold $\pi_D \underline{e} I$ is the amount of equity needed to fund all dirty firms.) A marginal increase in the capital require-

ment for dirty firms does not reverse the banks' ranking based on $r_q^{\max}$ and, thus, crowds out dirty lending at the margin.

1.b If $E \in (\pi_D \underline{e}I, \underline{e}I)$, all dirty firms are inframarginal and the clean firm type is marginal (not all clean types can be financed). Therefore, an increase in the capital requirement for (inframarginal) dirty firms reduces the equity that can be allocated to fund clean firms. Lending to clean firms is crowded out.

1.c if $E > \underline{e}I$, the banking sector has excess equity relative to the funding needs of all firms, $\underline{e}I$. Marginal changes in capital requirements have no effect.

2. Given any brown penalizing factor Δ_{BPF} that satisfies $r_D^{\max}(\underline{e} + \Delta_{BPF}) < r_C^{\max}(\underline{e})$, clean firms rank above dirty firms. (Such a BPF always exists by our assumption that the productivity of clean and dirty firms is sufficiently close.) As a result, dirty firms will only get bank funding after all lending opportunities to clean firms are exhausted. This increases lending to clean firms (strictly so if they were rationed before) and decreases lending to dirty firms (strictly so if bank capital is scarce after the increase in capital requirements for dirty firms).

■

Proof of Proposition 2: The proof is analogous to the proof of Proposition 1. ■

Lemma A.1 *Suppose a borrower's cash flow distribution is log-normal with mean cash flow $X_q(s)$ and return volatility σ. Then, if this borrower is funded by a bank in an optimal portfolio (see Result 1), the value of the deposit insurance put is given by:*

$$PUT_q\left(\underline{e}_q\right) = N\left(-d_2\right)\left(1-\underline{e}_q\right)I - N\left(-d_1\right)\overline{X}_q, \tag{A.8}$$

$$d_1 = \frac{\ln\left(\overline{X}_q\right) - \ln\left(I\left(1-\underline{e}_q\right)\right)}{\sigma} + \frac{\sigma}{2}, \tag{A.9}$$

$$d_2 = \frac{\ln\left(\overline{X}_q\right) - \ln\left(I\left(1-\underline{e}_q\right)\right)}{\sigma} - \frac{\sigma}{2}, \tag{A.10}$$

where N denotes the standard normal cumulative distribution function.

Proof of Lemma A.1: The Black-Scholes formula (see e.g., Hull (2003)) states that the value of a put option on an asset with price S and volatility σ, given a strike price K, option maturity T, and risk-free rate r, is given by

$$P = e^{-rT}KN\left(-d_2\right) - SN\left(-d_1\right), \tag{A.11}$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_2 + \sigma\sqrt{T}. \tag{A.12}$$

Risk-neutrality and zero discounting imply that in our setting $S = \overline{X}_q$. The strike price of the put option generated by deposit insurance is $K = I\left(1 - \underline{e}_q\right)$. This yields Equations (A.8), (A.9), and (A.10). ■

Lemma A.2 *The following comparative statics of the put value apply:*

$$\frac{\partial PUT_q}{\partial \sigma} > 0 \tag{A.13}$$

$$\frac{\partial PUT_q}{\partial \overline{X}_q} = -N\left(-d_1\right) < 0 \tag{A.14}$$

$$\frac{\partial PUT_q}{\partial \bar{e}_q} = -I \cdot \left(1 - N\left(d_2\right)\right) < 0 \tag{A.15}$$

$$\frac{\partial^2 PUT_q}{\partial \bar{e}_q^2} = IN'\left(d_2\right)\frac{1}{\sigma}\frac{1}{1-\underline{e}_q} > 0 \tag{A.16}$$

$$\frac{\partial^2 PUT_q}{\partial \bar{e}_q \partial \overline{X}_q} = IN'\left(d_2\right)\frac{1}{\sigma}\frac{1}{\overline{X}_q} > 0 \tag{A.17}$$

$$\frac{\partial^2 PUT_q}{\partial \bar{e}_q \partial \sigma} = -IN'\left(d_2\right)\left(\frac{\ln\left(\overline{X}_q\right) - \ln\left(I\left(1-\underline{e}_q\right)\right)}{\sigma^2} + \frac{1}{2}\right) < 0 \tag{A.18}$$

Proof: The first three results are standard (see, e.g., Hull (2003)). To show the remaining results, it is useful to write

$$\frac{\partial PUT_q}{\partial \bar{e}_q} = -I + IN\left(d_2\right). \tag{A.19}$$

Since $d_2 = \frac{\ln\left(\overline{X}_q\right) - \ln\left(I\left(1-\underline{e}_q\right)\right)}{\sigma} - \frac{\sigma}{2}$, see (A.10), we obtain

$$\frac{\partial d_2}{\partial \bar{e}_q} = \frac{1}{\sigma}\frac{1}{1-\underline{e}_q} > 0, \tag{A.20}$$

$$\frac{\partial d_2}{\partial \overline{X}_q} = \frac{1}{\sigma}\frac{1}{\overline{X}_q} > 0, \tag{A.21}$$

$$\frac{\partial d_2}{\partial \sigma} = -\left(\frac{\ln\left(\frac{\overline{X}_q}{I\left(1-\underline{e}_q\right)}\right)}{\sigma^2} + \frac{1}{2}\right) < 0. \tag{A.22}$$

Using $\frac{\partial^2 PUT_q}{\partial \bar{e}_q^2} = IN'\left(d_2\right)\frac{\partial d_2}{\partial \bar{e}_q}$ and (A.20), we obtain (A.16) and, analogously, (A.17) and (A.18). Note that (A.22) is unambiguously negative because both projects are, by assumption, positive NPV from a financial perspective, i.e., $\overline{X}_q > I > I\left(1-\underline{e}_q\right)$, and $\ln\left(x\right) > 0$ for any $x > 1$. ■

Lemma A.3 *If* $\lambda > \frac{NPV_q}{PUT_q(0)}$, *the maximizer of* $PPI_q(\underline{e}_q) = \frac{NPV_q - \lambda \cdot PUT_q(\underline{e}_q)}{I\underline{e}_q}$ *is finite and uniquely determined by the first-order condition*

$$IPPI_q(\underline{e}_q) = -\lambda \frac{\partial PUT_q(\underline{e}_q)}{\partial \underline{e}_q}. \tag{A.23}$$

Proof of Lemma A.3: The first-order condition $\frac{\partial PPI_q(\underline{e}_q)}{\partial \underline{e}_q} = 0$ implies

$$\frac{I\underline{e}_q \left(-\lambda \cdot \frac{\partial \mathrm{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q}\right) - \left(\mathrm{NPV}_q - \lambda \cdot \mathrm{PUT}_q(\underline{e}_q)\right) I}{I^2 \underline{e}_q^2} = 0. \tag{A.24}$$

Rearranging yields (A.23). To prove uniqueness, it is useful to rewrite (A.24) as

$$G\left(\underline{e}_q\right) = \mathrm{NPV}_q, \tag{A.25}$$

where the function

$$G\left(\underline{e}_q\right) := \lambda \left[\mathrm{PUT}_q(\underline{e}_q) - \underline{e}_q \frac{\partial \mathrm{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q}\right] \tag{A.26}$$

is defined on the domain $[0, 1]$. It is now easy to verify that the function G takes on its maximum value at 0 with $G(0) = \lambda \mathrm{PUT}_q(0) > 0$ and the minimum value at 1 with $G(1) = 0$. Moreover, G is differentiable and strictly decreasing with slope

$$G'\left(\underline{e}_q\right) = \lambda \left[\frac{\partial PUT}{\partial \underline{e}} - \left(\frac{\partial PUT}{\partial \underline{e}} + \underline{e} \frac{\partial^2 PUT}{\partial \underline{e}^2}\right)\right] = -\lambda \underline{e} \frac{\partial^2 PUT}{\partial \underline{e}^2} < 0, \tag{A.27}$$

where the last inequality uses $\frac{\partial^2 PUT}{\partial \underline{e}^2} > 0$, see (A.16). Since G is strictly decreasing and $G(1) = 0 < \mathrm{NPV}_q$, (A.25) has a solution if and only if $G(0) > \mathrm{NPV}_q$, which is equivalent to $\lambda > \frac{\mathrm{NPV}_q}{PUT_q(0)}$. By continuity of G, the solution for $\underline{e}_q$ is unique. ■

Proof of Proposition 3: We prove each claim separately.

P1 We first prove that, under optimal prudential regulation, it is without loss of generality to restrict dividends to zero.

First, suppose that, at the optimal capital requirements $\underline{\mathbf{e}}^*$, banks earn a scarcity rent (i.e., $r_E^* > 0$). In this case, banks strictly prefer not to pay out dividends, since they can earn an excess return.

Second, consider the case in which bank equity is not scarce, so that all types are funded, $\omega_q\left(\underline{\mathbf{e}}^*\right) = \pi_q$, and banks do not earn a scarcity rent (i.e., $r_E^* = 0$). In this case the regulator's payoff is given by:

$$\sum \pi_q \left[\mathrm{NPV}_q - \lambda \cdot \mathrm{PUT}_q(\underline{e}_q^*)\right]. \tag{A.28}$$

Now suppose (by contradiction) that under optimal prudential regulation not all equity is used, $E - \sum \pi_q \underline{e}_q^* I > 0$, so that the banking sector finds it optimal to pay

out the excess equity as dividends (as to ensure maximal leverage, see Result 1). Then the regulator could increase capital requirements for both types to $\underline{\mathbf{e}}^{**} > \underline{\mathbf{e}}^{*}$ (where the inequality is strict for at least one type) until all equity is exhausted (i.e., $E = \sum \pi_q \underline{e}_q^{**} I$). By construction, this would leave firm funding unaffected and strictly reduce the value of the deposit insurance put, thereby increasing the regulator's payoff. Hence, $\underline{\mathbf{e}}^{*}$ could not have been optimal.

We now turn to the remaining claims. It is useful to phrase the regulator's problem in terms of the $\text{PPI}_q(\underline{e}_q) = \frac{\text{NPV}_q - \lambda \cdot \text{PUT}_q(\underline{e}_q)}{I\underline{e}_q}$ (see Lemma A.3), and to denote the fraction of equity allocated to type q by $\tilde{\omega}_q$.

Problem 1 *The prudential regulator solves:*

$$\max_{\underline{\mathbf{e}}} \Omega_P = E \max_{\underline{\mathbf{e}}} \sum \tilde{\omega}_q \left(\underline{\mathbf{e}}\right) PPI_q(\underline{e}_q), \tag{A.29}$$

s.t. to a short-selling constraint (i.e., the equity allocated to each type is non-negative),

$$\tilde{\omega}_q \left(\underline{\mathbf{e}}\right) \geq 0, \tag{A.30}$$

the constraint that the mass of funded firms cannot exceed the supply of each type π_q,

$$\tilde{\omega}_q \left(\underline{\mathbf{e}}\right) E \leq \pi_q \underline{e}_q I, \tag{A.31}$$

and the incentive constraint governing the banking sector's privately optimal allocation of equity,

$$\tilde{\omega}_q \left(\underline{\mathbf{e}}\right) = \frac{\min \left\{ \max \left\{ E - \sum_{\check{q}: r_{\check{q}}^{\max} > r_q^{\max}} \pi_{\check{q}} \underline{e}_{\check{q}} I, 0 \right\}, \pi_q \underline{e}_q I \right\}}{E}. \tag{IC}$$

(IC) fully determines the funding decisions of the banking sector based on the ranking implied by $r_q^{\max}$.[16] For any given type q, the equity left after funding all types with higher ROE is given by $\max \left\{ E - \sum_{\check{q}: r_{\check{q}}^{\max} > r_q^{\max}} \pi_{\check{q}} \underline{e}_{\check{q}} I, 0 \right\}$. The actual amount allocated to a given type is then the minimum of the residual equity for this type, $\max \left\{ E - \sum_{\check{q}: r_{\check{q}}^{\max} > r_q^{\max}} \pi_{\check{q}} \underline{e}_{\check{q}} I, 0 \right\}$, and the amount of equity needed to fund all firms of type q, $\pi_q \underline{e}_q I$. As is now clear, banks' optimal decisions according to (IC) automatically ensure that the constraints (A.30) and (A.31) are satisfied. However, it is still useful to add these constraints to prove Principles P2 to P4.

[16] Our assumptions ensure that $r_q^{\max} > 0$ for all types q. If this were not the case, we would obtain $\tilde{\omega}_q \left(\underline{\mathbf{e}}\right) = 0$ for all types with $r_q^{\max} \leq 0$.

P2 Consider the regulator's relaxed problem, in which (A.31) and (IC) are ignored. This relaxed problem provides an upper bound to the regulator's payoff. In this relaxed problem, the regulator simply maximizes the convex combination of prudential profitability indices, $\sum \tilde{\omega}_q(\underline{\mathbf{e}})\, PPI_q(\underline{e}_q)$. The optimal choice is given by allocating all equity to the regulator's preferred type $\hat{q}$ (see Definition 1), which offers the maximal PPI, $\max_q \text{PPI}_q(\underline{e}_q^{PPI})$, yielding a total payoff of $E \max_q \text{PPI}_q(\underline{e}_q^{PPI}) = E \cdot \text{PPI}_{\hat{q}}(\underline{e}_{\hat{q}}^{PPI})$. (Our assumptions ensure that $\max_q \text{PPI}_q(\underline{e}_q^{PPI})$ is finite, see proof of Lemma A.3.) In our baseline model, the dirty type is the preferred type (i.e., $\hat{q} = D$) because dirty firms have a higher NPV and a lower put (for any given level of capital requirements). We now prove that the regulator can achieve this upper bound payoff in the full problem (i.e., after including constraints (IC) and (A.31)) if and only if the equity needed to fund all firms of the preferred type, $\pi_{\hat{q}} \underline{e}_{\hat{q}}^{PPI} I$, is greater than the supply of bank equity, i.e., $\pi_{\hat{q}} \underline{e}_{\hat{q}}^{PPI} I > E$. To see this, set $\underline{e}_{\hat{q}} = \underline{e}_{\hat{q}}^{PPI}$ (corresponding to $\underline{e}_D = \underline{e}_D^{PPI}$ in our baseline setting) and $\underline{e}_q = 1$ for all types with $q \neq \hat{q}$ (i.e., $\underline{e}_C = 1$ in our baseline setting). Given these capital requirements, banks rank type $\hat{q}$ highest (i.e., $r_{\hat{q}}^{\max}\left(\underline{e}_{\hat{q}}^{PPI}\right) > r_q^{\max}(1)$) so that (IC) implies that banks optimally allocate all equity to firm type $\hat{q}$, $\tilde{\omega}_{\hat{q}} = \frac{\min\left\{E, \pi_{\hat{q}} \underline{e}_{\hat{q}}^{PPI} I\right\}}{E} = 1$. To see why banks rank type $\hat{q}$ highest, note that

$$\begin{aligned} r_{\hat{q}}^{\max}\left(\underline{e}_{\hat{q}}^{PPI}\right) &= \frac{\text{NPV}_{\hat{q}} + \text{PUT}_{\hat{q}}\left(\underline{e}_{\hat{q}}\right)}{I \underline{e}_{\hat{q}}} \\ &> \text{PPI}_{\hat{q}}(\underline{e}_{\hat{q}}) = \frac{\text{NPV}_{\hat{q}} - \lambda \text{PUT}_{\hat{q}}\left(\underline{e}_{\hat{q}}\right)}{I \underline{e}_{\hat{q}}} \\ &> \max_{\underline{e}_q} \text{PPI}_q(\underline{e}_q) \\ &\geq \text{PPI}_q(1) = \frac{\text{NPV}_q}{I} = r_q^{\max}(1), \end{aligned}$$

where line 2 follows from the fact that the put is positive, and line 3 follows from the fact that $\hat{q}$ (rather than q) maximizes the PPI. Line 4 follows because the maximized value of the PPI must exceed $\text{PPI}_q(1) = \frac{\text{NPV}_q}{I}$, which is also the maximal ROE for type q if $\underline{e}_q = 1$. As a result, $r_{\hat{q}}^{\max}\left(\underline{e}_{\hat{q}}^{PPI}\right) > r_q^{\max}(1)$ and (A.31) is slack.

P3 Suppose that type q_M is marginal, i.e., $0 < \tilde{\omega}_{q_M}(\underline{\mathbf{e}}) < \frac{\pi_{q_M} \underline{e}_{q_M} I}{E}$. Then (A.29) and (IC) imply that the regulator's payoff is given by

$$\sum_{q: r_q^{\max} > r_{q_M}^{\max}} \pi_q \left[\text{NPV}_q - \lambda \cdot \text{PUT}_q(\underline{e}_q)\right] + \left(E - \sum_{q: r_q^{\max} > r_{q_M}^{\max}} \pi_q \underline{e}_q I\right) \text{PPI}_{q_M}(\underline{e}_{q_M}). \tag{A.32}$$

It is now easy to see that optimality of $\underline{e}_{q_M}$ requires that $\underline{e}_{q_M}^* = \arg\max_{\underline{e}_{q_M}} \text{PPI}_{q_M}\left(\underline{e}_{q_M}\right)$, since all other terms are independent of $\underline{e}_{q_M}$. This maximizer exists by Lemma A.3 and satisfies the first-order condition (A.23).

P4 We have to consider two cases. First, consider the case, in which all profitable types

are financed. Then, the regulator's objective is

$$\sum_{q:r_q^{\max}>0} \pi_q \left[\mathrm{NPV}_q - \lambda \cdot \mathrm{PUT}_q(\underline{e}_q)\right], \tag{A.33}$$

s.t. to the (binding) equity capacity constraint (by Principle P1)

$$E - \sum \pi_q \underline{e}_q I = 0. \tag{A.34}$$

(A.33) is a concave objective subject to a linear constraint (A.34). Denoting the associated Lagrange multiplier by κ, we obtain the necessary and sufficient optimality condition

$$-\lambda \pi_q \frac{\partial \mathrm{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q} = \kappa \pi_q I, \tag{A.35}$$

which means that the marginal put value for all types is a constant,

$$-\frac{\partial \mathrm{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q} = \frac{\kappa I}{\lambda}, \tag{A.36}$$

implying (14).
Next suppose that not all types are fully financed, i.e., there is a marginal firm type $0 < \tilde{\omega}_{q_M}(\underline{\mathbf{e}}) < \frac{\pi_{q_M} \underline{e}_{q_M} I}{E}$. Then for all inframarginal types q, the first-order condition of (A.32) implies:

$$-\pi_q I \mathrm{PPI}_{q_M}(\underline{e}_{q_M}) = \lambda \pi_q \frac{\partial \mathrm{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q}. \tag{A.37}$$

Since the marginal type's capital requirement maximizes its PPI, we obtain, using (A.23), that

$$I \mathrm{PPI}_{q_M}(\underline{e}_{q_M}) = -\lambda \frac{\partial \mathrm{PUT}_{q_M}(\underline{e}_{q_M})}{\partial \underline{e}_{q_M}}. \tag{A.38}$$

Taken together, the two first-order conditions (A.37) and (A.38) imply that the marginal puts are equalized

$$\frac{\partial \mathrm{PUT}_{q_M}(\underline{e}_{q_M})}{\partial \underline{e}_{q_M}} = \frac{\partial \mathrm{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q}. \tag{A.39}$$

■

Proof of Proposition 4: Marginal changes in the cash flow distribution do not affect the regulator's ranking. This implies that, in our setting, dirty firms continue to rank above clean firms. The proof below is written so that its claims can be applied to changes in the cash flow distribution of either type.

1. Suppose firm type q is marginal, then its capital requirement is only a function of its own cash-flow distribution characterized by $\left(\overline{X}_q, \sigma_q\right)$ and satisfies, by principle

P3 (see Proposition 3) and Proof of Lemma A.3, the first-order condition

$$\overline{X}_q - I - G\left(\underline{e}_q\right) = 0, \tag{A.40}$$

where

$$G\left(\underline{e}_q\right) := \lambda\left[\mathrm{PUT}_q(\underline{e}_q) - \underline{e}_q \frac{\partial \mathrm{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q}\right]. \tag{A.41}$$

Since $G'\left(\underline{e}_q\right) < 0$ (see Proof of Lemma A.3), we obtain that $\left|G'\left(\underline{e}_q\right)\right| = -G'\left(\underline{e}_q\right)$. The comparative statics of the marginal type now follow from applying the implicit function theorem to (A.40),

$$\frac{\partial \underline{e}_q}{\partial \overline{X}_q} = -\frac{1 - \frac{\partial G\left(\underline{e}_q\right)}{\partial \overline{X}_q}}{\left|G'\left(\underline{e}_q\right)\right|} < 0, \tag{A.42}$$

$$\frac{\partial \underline{e}_q}{\partial \sigma_q} = \frac{\frac{\partial G\left(\underline{e}_q\right)}{\partial \sigma_q}}{\left|G'\left(\underline{e}_q\right)\right|} > 0, \tag{A.43}$$

where

$$\frac{\partial G\left(\underline{e}_q\right)}{\partial \overline{X}_q} = \lambda\left[\frac{\partial \mathrm{PUT}_q(\underline{e}_q)}{\partial \overline{X}_q} - \underline{e}_q \frac{\partial^2 \mathrm{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q \partial \overline{X}_q}\right] < 0, \tag{A.44}$$

$$\frac{\partial G\left(\underline{e}_q\right)}{\partial \sigma_q} = \lambda\left[\frac{\partial PUT_q}{\partial \sigma} - \underline{e}_q \frac{\partial^2 \mathrm{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q \partial \sigma_q}\right] > 0. \tag{A.45}$$

The respective signs follow directly from Lemma A.2.

2. In region 2, where one type is fully financed and the other type is not financed (see Figure B.1), the capital requirement is just a function of the supply constraint,

$$\underline{e}_q = \frac{E}{\pi_q I}, \tag{A.46}$$

which is independent of $\overline{X}_q$ and σ_q.

3. If both types are financed (see, e.g., regions 3 or 4 in Figure B.1), the first-order conditions imply that marginal puts are equalized,

$$\frac{\partial PUT_q}{\partial \underline{e}_q} - \frac{\partial PUT_{\tilde{q}}}{\partial \underline{e}_{\tilde{q}}} = 0. \tag{A.47}$$

Since $\frac{\partial^2 PUT_q}{\partial \underline{e}_q^2} > 0$ (by Lemma A.2), we obtain the following comparative statics

with respect to changes in the own cash flow characteristics:

$$\frac{\partial \underline{e}_q}{\partial \overline{X}_q} = -\frac{\frac{\partial^2 PUT_q}{\partial \underline{e}_q \partial X_q}}{\frac{\partial^2 PUT_q}{\partial \underline{e}_q^2}} < 0, \tag{A.48}$$

$$\frac{\partial \underline{e}_q}{\partial \sigma_q} = -\frac{\frac{\partial^2 PUT_q}{\partial \underline{e}_q \partial \sigma_q}}{\frac{\partial^2 PUT_q}{\partial \underline{e}_q^2}} > 0, \tag{A.49}$$

where $\frac{\partial^2 PUT_q}{\partial \underline{e}_q \partial X_q} > 0$ (by Lemma A.2) and $\frac{\partial^2 PUT_q}{\partial \underline{e}_q \partial \sigma_q} < 0$ (by Lemma A.2). The comparative statics regarding changes in the cash flow distribution of the other type $\tilde{q}$ satisfy:

$$\frac{\partial \underline{e}_q}{\partial \overline{X}_{\tilde{q}}} = \frac{\frac{\partial^2 PUT_{\tilde{q}}}{\partial \underline{e}_{\tilde{q}} \partial X_{\tilde{q}}}}{\frac{\partial^2 PUT_q}{\partial \underline{e}_q^2}} > 0, \tag{A.50}$$

$$\frac{\partial \underline{e}_q}{\partial \sigma_{\tilde{q}}} = \frac{\frac{\partial^2 PUT_{\tilde{q}}}{\partial \underline{e}_{\tilde{q}} \partial \sigma_{\tilde{q}}}}{\frac{\partial^2 PUT_q}{\partial \underline{e}_q^2}} < 0. \tag{A.51}$$

Taking together all these cases, we obtain the claims in Proposition 4. ■

Proof of Proposition 5: Recall the definition of the PPI,

$$\text{PPI}_q(\underline{e}_q^{PPI}) = \max_{\underline{e}_q} \frac{\overline{X}_q - I - \lambda \cdot \text{PUT}_q(\underline{e}_q)}{I\underline{e}_q}.$$

We first consider changes in the mean payoff $\overline{X}_q$. As is immediate, the PPI is strictly increasing in $\overline{X}_q$ (both because of higher financial NPV and lower put value). Moreover, if $\overline{X}_q < I$, the $\text{PPI}_q(\underline{e}_q^{PPI})$ is below 0. As a result, there exists a threshold for $\overline{X}_D$ such that, for any $\overline{X}_D$ below that threshold, the PPI of the dirty technology is below the clean technology, which satisfies $\text{PPI}_C(\underline{e}_C^{PPI}) > 0$. That is, the regulator's preferred types switches.

We now consider changes in the volatility (holding the mean payoff $\overline{X}_q$ constant). Similarly, the PPI is strictly decreasing in σ_q. The lowest possible PPI is achieved, as volatility approaches infinity (which is optimally countered by $\lim_{\sigma_q \to \infty} \underline{e}_q^{PPI} = 1$). Hence,

$$\lim_{\sigma_q \to \infty} \text{PPI}_q(\underline{e}_q^{PPI}) = \lim_{\sigma_q \to \infty} \max_{\underline{e}_q} \frac{\overline{X}_q - I - \lambda \cdot \text{PUT}_q(\underline{e}_q)}{I\underline{e}_q} \tag{A.52}$$

$$= \frac{\overline{X}_q - I - \lambda \cdot \text{PUT}_q(1)}{I} \tag{A.53}$$

$$= \frac{\overline{X}_q - I}{I} = \frac{\text{NPV}_q}{I}. \tag{A.54}$$

Then, as long as $\frac{\text{NPV}_D}{I} < \text{PPI}_C(\underline{e}_C^{PPI})$, a change in the regulator's preferred type occurs if σ_D becomes sufficiently large. ■

Proof of Proposition 6: We first prove part 1 of the proposition. By definition of $\underline{e}_C^{IC}$, the bank regulator must set $\underline{e}_C \leq \underline{e}_C^{IC}$ to ensure that banks rank the clean type first. Now suppose that $E > \pi_C \underline{e}_C^{IC} I$. Then, after financing all clean types at a capital requirements of $\underline{e}_C \leq \underline{e}_C^{IC}$, the banking sector has $E - \pi_C \underline{e}_C^{IC} I$ of equity left over. Since dirty types are, by assumption, profitable at capital requirements of 100%, $r_D^{\max}(1) > 0$, banks use the residual equity to finance dirty firms.
We now prove part 2 of the proposition. If $E < \pi_C \underline{e}_C^{SPI} I$ and $\underline{e}_C \leq \underline{e}_C^{IC}$, the banking sector ranks clean firms first, and bank equity is not sufficient to fund all clean firms. As a result, no dirty firms are funded. This proves the first sentence of the second statement. Recall that $\underline{e}_C^{SPI}$ is defined as the capital requirement that maximizes the SPI. This would also be the prudentially optimal capital requirement if there were only clean types in the economy (given that $\phi_C = 0$, which implies that $\text{SPI}_C = \text{PPI}_C$). If $\underline{e}_C^{SPI} > \underline{e}_C^{IC}$, setting $e = \underline{e}_C^{SPI}$ would violate the banks' incentive constraint: Banks would prefer lending to dirty firms, as $r_D^{\max}(1) > r_C^{\max}\left(\underline{e}_C^{SPI}\right)$. To ensure that clean types are funded first, the regulator must then lower the capital requirement for clean firms to $\underline{e}_C^{IC} < \underline{e}_C^{SPI}$. This implies that capital requirements for clean firms are below the prudentially optimal level.
■

B Prudential Capital Requirements: An Illustration

Based on the four principles outlined in Proposition 3, Figure B.1 highlights four distinct regions linking optimum prudential capital regulation to the capitalization of the banking sector E. The lower panel plots optimal prudential capital requirements and the upper panel plots the corresponding funding decisions ω of the banking sector.

For sufficiently scarce equity, $E < E_1 := \pi_D \underline{e}_D^{\text{PPI}} I$, only the regulator's preferred (dirty) type is funded. Since the dirty type is partially funded, $\omega_D < \pi_D$ (see upper panel), Principle P3 applies and the optimum prudential capital requirement maximizes PPI_D, $\underline{e}_D^* = \underline{e}_D^{\text{PPI}}$. To ensure that banks find it incentive compatible to exclusively lend to dirty firms, capital requirements for the (unfunded) clean type must be set sufficiently high so as to ensure that $r_C^{\max}(\underline{e}_C^*) \leq r_D^{\max}(\underline{e}_D^*)$. This incentive constraint imposes a lower bound on the capital requirement for clean loans, which is illustrated by the dotted green line. As equity in the banking sector increases, the mass of funded dirty firms increases accordingly, as illustrated in the upper panel of Figure B.1. Once $E = E_1$, all dirty firms are funded, $\omega_D = \pi_D$.

In the second region, $E \in (E_1, E_2)$, dirty firms are fully funded, $\omega_D = \pi_D$. However, clean firms remain unfunded (see upper panel). Intuitively, once all dirty lending opportunities in the economy have been exhausted, the marginal benefit of funding the next best investment opportunity, the clean type, is lower by a discrete amount. Therefore, it is initially optimal to use additional equity in the banking sector to reduce the deposit insurance put associated with funding dirty firms, rather than inducing banks to fund clean firms. In this region, Principle P1 pins down optimal prudential capital

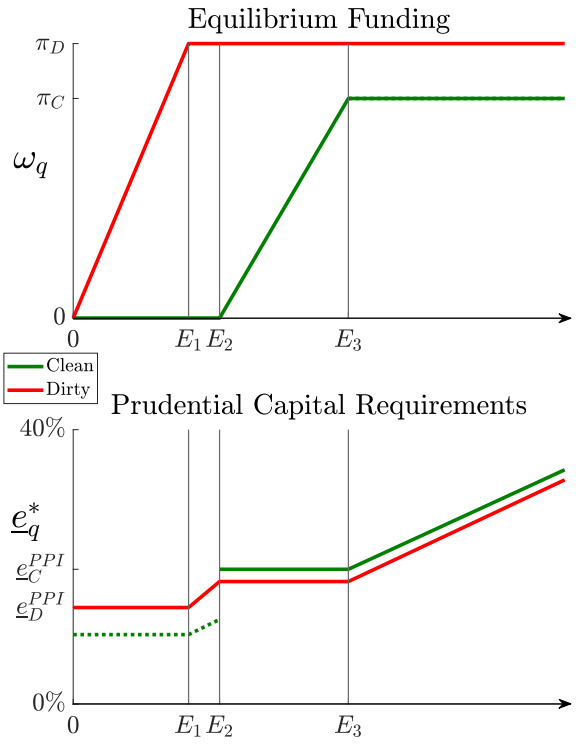

Figure B.1. Optimal prudential capital regulation. This figure plots equilibrium funding decisions (upper panel) and capital requirements (lower panel) under optimal prudential bank capital regulation. Clean firms and their capital requirements are plotted in green, dirty in red. The dotted green line indicates the lower bound on the capital requirement for (unfunded) clean firms in the region in which only dirty loans are funded and the capital requirement for dirty loans is set to $\underline{e}^*_D$. This lower bounds satisfies $r^{\max}_C(\underline{e}_C) = r^{\max}_D(\underline{e}^*_D)$.

requirements for the dirty type, $\underline{e}^*_D = \frac{\pi_D E}{I}$ (see lower panel).[17]

Once the capitalization of the banking sector reaches $E = E_2$, the marginal reduction in the deadweight cost associated with the deposit insurance put is equal to the marginal value of funding a clean firm. Therefore, in region 3, $E \in (E_2, E_3)$, it becomes optimal to induce banks to fund clean firms, which then become the marginal type. The clean capital requirement is, therefore, pinned down by Principle P3, so that $\underline{e}^*_C = \underline{e}^{PPI}_C$. The capital requirement for the inframarginal dirty type is determined by Principle P4, the equalization of marginal puts.

Finally, in region 4, $E > E_3$, both types are fully funded $\omega_q = \pi_q$ (see upper panel). In this region, banking sector equity does not affect production decisions in the economy. Therefore, any additional equity is used to phase out the deadweight costs arising from deposit insurance. Principles P1 and P4 jointly pin down the optimal capital requirements for clean and dirty types.

[17] Optimum clean capital requirements are again set to ensure that $r^{\max}_C(\underline{e}^*_C) < r^{\max}_D(\underline{e}^*_D)$, see green dotted line in the lower panel. Because dirty capital requirements are increasing in E in this region, the lower bound for clean capital requirement is increasing too to satisfy the bankers' IC constraint.