# Subtle Discrimination 

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# Subtle Discrimination* 

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#### Abstract

We introduce the concept of subtle discrimination-biased acts that cannot be objectively ascertained as discriminatory-and study its implications in a model of competitive promotions. When choosing among similarly qualified candidates, a principal with a subtle bias towards a particular group may plausibly deny being biased. We show that subtle (as opposed to overt) discrimination has unique implications. Discriminated candidates perform better in low-stakes careers, while favored candidates perform better in high-stakes careers. In equilibrium, firms are polarized: high-productivity firms become "progressive" and have diverse management teams, while low-productivity firms choose to be "conservative" and homogeneous at the top.


Keywords: Bias, human capital, promotions, firm performance
JEL Classification: M51, J71, J31

[^0]
## 1 Introduction

Economists traditionally classify discriminatory acts based on their source. Some view such acts as a consequence of rational statistical discrimination (Phelps (1972); Arrow (1973)). A second view is that biases in tastes, beliefs, or incentives can cause discrimination (Becker (1957); Bordalo et al. (2016); Bohren et al. (2019); Dobbie et al. (2021)). While such a classification is undoubtedly useful, other perspectives are also possible. Social and organizational psychologists classify discriminatory acts based on their transparency, i.e., whether discrimination is overt or subtle. These scholars define subtle discrimination as acts that are ambiguous in intent to harm, ex-post rationalizable, difficult to verify, and often (but not always) unintentional. ${ }^{1}$ In the workplace, examples include asking female employees to perform menial tasks, failing to praise the performance of minority employees, and disproportionately promoting men to managerial positions when choosing among equally qualified candidates.

It is hard to substantiate claims of subtle discrimination. In the United States, to prosecute a person or company for discrimination, the discriminated party must either provide direct evidence of intent to harm or deny rights or prove a clear pattern of adverse events that can be explained only by discrimination. Partially due to the threat of legal action, overt discrimination has become relatively rare. In contrast, subtle discrimination often hides behind non-discriminatory narratives, making it a common but invisible phenomenon.

Despite its prevalence, the impact of subtle discrimination on workers and firms has received scant attention in the economics and finance literatures. Our paper is a first attempt at formalizing the concept of subtle discrimination. We define subtle discrimination as biased acts that cannot be objectively ascertained as discriminatory. The defining feature of subtle discrimination is its plausible deniability. For instance, when a biased decision-maker acts in a discriminatory way, he or she may offer an explanation for such actions, such as using subjective criteria like "potential"

[^1]to choose between two candidates with similar objective qualifications, thereby concealing a bias against one of the candidates. When such a narrative is plausible, the decision-maker is subtly biased.

We apply our notion of subtle discrimination to a model of promotions. In the model, two exante identical agents with labels "blue" and "red" compete for promotion by investing in human capital. Labels are payoff-irrelevant; profit depends only on the promoted agent's acquired skills. The decision-maker (the principal) has a small bias in favor of the blue agent. Because the bias is small, the principal prefers to promote the red agent when her objective qualifications are considerably better than those of the blue agent. That is, the principal does not overtly discriminate. However, when both candidates are similarly qualified, the principal is more likely to promote the blue candidate. This form of discrimination is subtle because the principal's "tie-breaking" rule is not observable, and any given promotion decision can be justified as non-biased.

Our analysis draws from the reality that choosing among candidates with similar objective qualifications is often challenging. In such cases, the principal is likely to rely on subjective assessments. Such discretion allows biases to influence choices. In an employment setting, Hoffman et al. (2018) show that biases, not superior information, explain most of the cases in which managers use discretion to select candidates. Using internal data from a large bank, Bircan et al. (2022) found that supervisors' discretion in job assignments explains most of the gender promotion gap, implying the existence of "subtle mechanisms that disadvantage women."

Our paper makes three primary contributions. First, we propose a formalization of discriminatory acts into two categories: subtle and overt. Second, in a model of promotions, we show that subtle and overt discrimination have different empirical predictions. Third, our model of subtle discrimination in promotions generates a rich set of empirical predictions relating firm characteristics to the performance of different groups of workers, the diversity of top management teams, and firms' choices of anti-discrimination policies.

We distinguish subtle discrimination from overt discrimination based on the ease of objectively ascertaining particular acts as discriminatory. When two candidates have similar qualifications,
passing over one candidate for a promotion is not clear evidence of discrimination. In our model, when blue and red agents are similarly skilled, the principal can use his private signals to rationalize the act of promoting the blue agent, both to others and to himself. In contrast, promoting an unskilled blue agent over a skilled red agent is clear evidence of discrimination; we classify such acts as overt discrimination. While our particular formalization necessarily leaves out many nuances present in real life, it is simple, intuitive, and tractable.

Although individual acts of subtle discrimination cannot be observed, the agents form beliefs about the principal's subtle bias. Such beliefs affect the agents' decisions to invest in human capital. Because promotions are competitive, subtle biases also affect how the agents react to each other's decisions. In equilibrium, agents differ in their investment decisions, which creates an achievement gap, i.e., a difference in accumulated human capital and qualifications. Two opposing forces contribute to the achievement gap. On the one hand, unfavored agents are discouraged from investing in human capital because they anticipate a low probability of promotion. We call this force the discouragement effect. On the other hand, an unfavored agent may choose to overinvest in skills in an attempt to separate herself from the favored agent. We call this force the overcompensation effect.

We show that the sign and the magnitude of the achievement gap depend on the stakes faced by the agents. The stakes matter because they differentially affect the agents' incentives to invest. When the net benefit from promotion is large - a high-stakes career path - the discouragement effect dominates, implying that favored (blue) agents invest more than unfavored (red) agents. In this case, the achievement gap is positive: favored agents have more visible achievements (e.g., better qualifications and performance records) than unfavored agents. In contrast, when the net benefit from promotion is small - a low-stakes career path - the overcompensation effect dominates and, thus, favored agents invest less than unfavored agents, leading to a negative achievement gap. We show that these results hinge crucially on discrimination being subtle instead of overt.

These results are helpful when interpreting the evidence on the professional advancement of women and minorities. Evidence that women have lower promotion rates in high-skilled occu-
pations can be found in Hospido et al. (2019) for central bankers, in Bosquet et al. (2019) for academic economists, in Azmat et al. (2020) for lawyers, and in Bircan et al. (2022) for bankers. Azmat et al. (2020) show that female associates in law firms invest less in the qualifications required for promotion (e.g., hours billed) than male associates. Hospido et al. (2019) and Bosquet et al. (2019) find that women are less likely to seek promotion in the first place. Similarly, Linos et al. (2023) find that Black-White promotion gaps in a professional services firm can be explained partly by Black employees receiving worse subjective evaluations when assigned to teams with more White coworkers. In contrast, Benson et al. (2021) find that women in management-track careers in retail have better (pre-promotion) performance than men. ${ }^{2}$ These facts are consistent with our prediction that discriminated groups are discouraged from investing in promotable tasks in high-stakes careers while being over-incentivized to undertake such investments in low-stakes careers.

Our model also predicts that, in high-stakes careers, differences in observable achievements (such as human capital, performance, experience, and effort) explain most of the promotion gap (i.e., the difference in promotion rates between groups). Because the promotion gap increases with the expected benefits of promotion, the model can also explain the evidence of increasing promotion gaps at the top of hierarchies, a fact that is known as the "leaky pipe" phenomenon (Lundberg and Stearns (2019); Sherman and Tookes (2022)).

In the model, firms can change their subtle biases through anti-discrimination (i.e., diversity, equity, and inclusion) policies. Firms can become more progressive (i.e., less biased) or conservative (i.e., more biased) at no direct cost. In equilibrium, firms become polarized. On one side, we have high-productivity firms offering high-stakes careers to their employees. Such firms choose to become progressive and, thus, have greater diversity in their top management teams. On the other side, we have low-productivity firms that offer low-stakes careers. Such firms choose to be conservative and, thus, have little diversity at the top. The model predicts that even small differences in firm productivity can account for large differences in corporate culture and top-level

[^2]diversity. Furthermore, market forces cannot eliminate such differences. Thus, our model provides novel empirical predictions relating firm quality to observed diversity metrics. Consistent with the model's predictions, Edmans et al. (2023) find that employees' perception of diversity, equity, and inclusion is stronger in growing, high-valuation, and financially strong firms.

Our model offers a novel perspective on the costs and benefits of corporate focus on social issues, especially regarding workforce diversity. While some argue that progressive values typically do not conflict with the pursuit of profits (e.g., Edmans (2020)), others claim that some businesses excessively focus on promoting progressive causes to the detriment of profits (see EdgecliffeJohnson (2022)). Our model illustrates one mechanism through which progressive firms can increase profits: Employees who believe that a company does not discriminate in promotions are encouraged to invest in promotable tasks. Moreover, the model shows that such benefits accrue primarily to firms with high returns to human capital. In contrast, firms in which investment in human capital has low returns have less to gain from eliminating discrimination in promotions. For such firms, investing in a reputation for progressiveness does not pay off.

In the next section, we define subtle discrimination in the context of a choice between two candidates for a position. We present our main analysis in Section 3. Section 4 reviews some of the related theoretical literature. We discuss the main empirical implications in Section 5 and conclude. The Online Appendix (OA) presents further discussions on testing and empirical relevance (Section OA.2), welfare and policy analyses, and several variations of our main model.

## 2 Definition and Interpretation

A decision-maker needs to choose one of two candidates $i \in\{b, r\}$, called Blue and Red. Blue and Red have objective qualifications - called skills - summarized by $s_{b}$ and $s_{r}$, which are observed by everyone. Without loss of generality, we set $\Delta s \equiv s_{r}-s_{b} \geq 0$. The decision-maker privately observes a (subjective) signal $x_{i}$ for each agent. The signals are independent and identically distributed random variables with support $[\underline{x}, \bar{x}]$. Let $F($.$) denote the cumulative distribution function$
of $\Delta x \equiv x_{r}-x_{b}$. The decision-maker has a mandate to maximize an observable payoff (e.g., profit), $\pi$. If the decision-maker chooses agent $i$, the payoff is $\pi_{i}=s_{i}+\omega x_{i}+u$, where $\omega>0$ and $u$ is a zero-mean random variable. We assume that everyone knows $\omega$ and holds the same beliefs about $F($.$) and the distribution of u$.

In the absence of biases, after observing $s_{i}$ and $x_{i}$, the decision-maker chooses Blue if $\Delta s+$ $\omega \Delta x<0$. Thus, the probability of an unbiased decision-maker choosing Blue is $F\left(-\frac{\Delta s}{\omega}\right)=$ $F\left(\frac{s_{b}-s_{r}}{\omega}\right)$. In the case of a biased decision-maker, to keep the analysis general, we model the decision-maker's behavior directly without specifying payoffs and beliefs; we discuss different microfoundations below. Let $P\left(s_{b}, s_{r}, \omega\right)$ denote the probability that the decision-maker chooses Blue given $s_{b}, s_{r}$ and $\omega$. We assume that $P\left(s_{b}, s_{r}, \omega\right)$ is increasing in $s_{b}$ and decreasing in $s_{r}$. While $P\left(s_{b}, s_{r}, \omega\right)$ cannot be directly observed, the candidates and other observers may form beliefs about it. We assume that such beliefs are correct to avoid introducing additional behavioral considerations.

We define the decision-maker's bias towards Blue as the excess probability of choosing Blue not justified by the qualification gap, $s_{b}-s_{r}$, and the importance of subjective signals for profit, $\omega$ :

$$
\begin{equation*}
b\left(s_{b}, s_{r}, \omega\right)=P\left(s_{b}, s_{r}, \omega\right)-F\left(\frac{s_{b}-s_{r}}{\omega}\right) \geq 0 \tag{1}
\end{equation*}
$$

There could be several sources for the bias in (1). For example, the decision-maker may prefer blue agents (as in Becker (1957)). Alternatively, the decision-maker may incorrectly believe that blue agents are more productive (Bordalo et al. (2016); Bohren et al. (2019)). Even with correct beliefs, the decision-maker may be better at reading subjective signals from blue agents (e.g., Cornell and Welch (1996); Fershtman and Pavan (2021)). Biases may also be caused by external factors, such as poorly designed incentive structures (Dobbie et al. (2021)). In what follows, we do not take a stand on whether biases are caused by beliefs or preferences, or whether they are intrinsic or extrinsic. Our model can accommodate most of these interpretations.

Next, we define subtle bias:

Definition. If $F\left(\frac{s_{b}-s_{r}}{\omega}\right)>0$, we say that bias $b\left(s_{b}, s_{r}, \omega\right)$ is subtle.

By assumption, Red's objective qualifications are (weakly) better than Blue's qualifications ( $\Delta s \geq$ $0)$. Nevertheless, the decision-maker may still choose Blue, either because the decision-maker is biased towards Blue or because the decision-maker privately observes signal $\Delta x$, such that $-\omega \Delta x \geq \Delta s$. That is, whenever there exist potential signal realizations that justify choosing Blue, i.e., $F\left(\frac{s_{b}-s_{r}}{\omega}\right)>0$, a biased decision-maker can "plausibly deny" being biased if only a small number of decisions are observed. Plausible deniability is the defining property of subtle discrimination. In our formalization, plausible deniability means that, if everyone observes $\Delta s$ and $\omega$ and has the same belief about $F($.$) , an act of choosing Blue does not constitute conclusive evidence$ that the decision-maker is biased. Subtle discrimination is thus an act of favored treatment of an agent or group of agents when the decision-maker can resort to a plausible-deniability defense of such an act.

In contrast with subtle biases, we now define overt bias:

Definition. If $F\left(\frac{s_{b}-s_{r}}{\omega}\right)=0$, we say that bias $b\left(s_{b}, s_{r}, \omega\right)$ is overt.

Case $F\left(\frac{s_{b}-s_{r}}{\omega}\right)=0$ occurs when $\Delta s>\omega(\bar{x}-\underline{x})$. In this case, a single act of choosing Blue is incontrovertible evidence of biased decision-making. While a binary classification of biases into subtle or overt is helpful, biases can also vary in "subtlety." In particular, without the bounded support assumption, all biases would be subtle. Still, a subtle bias with very small $F\left(\frac{s_{b}-s_{r}}{\omega}\right)$ is, for all practical purposes, an overt bias, because one would find it difficult to defend choosing Blue. We can thus think of $F\left(\frac{s_{b}-s_{r}}{\omega}\right)$ as a measure of subtlety in discrimination. Bias subtlety is maximized at $s_{b}=s_{r}$ (recall that $s_{b} \leq s_{r}$ by assumption). Thus, subtle discrimination is most likely to occur when objective differences are small. When agents are observationally equivalent, any choice is rationalizable, even when the importance of subjective information is minimal (i.e., when $\omega \rightarrow 0)$.

By definition, an individual act of subtle discrimination is not immediately detectable. While outcome realizations (i.e., $\pi$ ) might be informative about underlying biases, they may not be suffi-
cient to reveal subtle biases because of the noise in performance $(u)$. In practice, even after observing a long sequence of decisions, reliably estimating subtle biases is challenging. First, for small differences in observables $\Delta s$, one would need a large sample to detect subtle discrimination with a reasonable degree of confidence. Second, for small differences $\Delta s$, even mild disagreements about $\omega$ and $F($.$) can provide sufficient cover for subtle biases. Thus, subtle discrimination is preva-$ lent in situations where (i) differences in objective qualifications between groups are small, (ii) decisions (like promotions) by a single decision-maker are infrequent, and (iii) observers disagree about the importance of subjective information for forecasting performance.

Our notion of subtle discrimination is also compatible with unconscious biases in decisionmaking. This interpretation is valid when the decision-maker does not directly benefit from choosing a particular type. One interpretation is that the decision-maker always rationalizes his choice (as in Cherepanov et al. (2013)). In practice, the decision-maker might find it difficult to correct the bias (at least in a finite series of decisions) if he believes that his choices are unbiased. Such unconscious biases are most likely to pertain to System 1 thinking, i.e., fast, automatic, and effortless associations (Kahneman (2011)).

While establishing the presence of subtle biases with confidence might be hard in practice, agents and observers may hold (correct or incorrect) beliefs about the existence of such biases. ${ }^{3}$ Thus, subtle biases may affect agents' decisions to invest in observable skills $\left(s_{i}\right)$. In our application below, we show that in competitive environments, subtle biases can be either attenuated or amplified and result in substantial differences in outcomes between groups.

## 3 A Model of Subtle Discrimination in Promotions

In this section, we present an application of our concept of subtle discrimination to a model of promotions. For simplicity, we consider the limiting case in which the principal's subjective information is negligible for the decision: $\omega \rightarrow 0$. In that case, subtle discrimination can occur only

[^3]when the difference in skills, $s_{r}-s_{b}=\Delta s$, is zero. After presenting the setup in Subsection 3.1, in Subsection 3.2, we describe the first-best solution to serve as a benchmark. We then solve the model for an exogenously given compensation contract in Subsection 3.3. In Subsection 3.4, we let firms choose the compensation contracts optimally. In Subsection 3.5, we endogenize the subtle bias. In the OA, we discuss the robustness of the model to several different assumptions.

### 3.1 Definitions and Model Setup

At Date 0, a firm hires two ex-ante identical agents - b (Blue) and $r$ (Red) - for an entry-level position (job 1). Red and Blue are payoff-irrelevant labels. The firm needs to fill both vacancies. We assume that the firm does not (or cannot) discriminate at the hiring stage; thus, the 50/50 split between $b$ and $r$ reflects the composition of the candidate pool. That is, we implicitly assume that some frictions prevent firms from picking only one type. ${ }^{4}$

At Date 1, the agents simultaneously undertake a non-observable investment (or effort), $e_{i} \in$ $[0,1], i \in\{b, r\}$, to acquire a "skill." The skill is an observable but not verifiable binary variable: $s_{i} \in\{0,1\} .{ }^{5}$ We interpret skill broadly as any kind of observable evidence that predicts an agent's future performance. We assume that the skill is firm-specific in the sense that it is less valuable to agents who leave the firm; see Sections OA. 4 and OA. 5 for model extensions in which agents invest in (partially or fully) general skills. Agent $i$ 's probability of acquiring the skill is $e_{i}$. Both agents are risk-neutral and have the same cost function, $c\left(e_{i}\right)$, which is strictly increasing, strictly convex, and satisfies $c(0)=0 .{ }^{6}$ That is, agent $i$ 's utility is $u_{i}=w_{i}-c\left(e_{i}\right)$, where $w_{i}$ is the agent's monetary compensation.

[^4]At Date 2, the principal can choose one of the agents to fill a top position (job 2). ${ }^{7}$ Agents who are not promoted remain in the entry-level job; we normalize the revenue generated by these to zero. Agents can be skilled $\left(s_{i}=1\right)$ or unskilled $\left(s_{i}=0\right)$. Promoting an unskilled agent increases the principal's expected payoff by $l>0$ while promoting a skilled agent increases the payoff by $l+H$, where $H>0$ denotes the productivity gain upon promotion of a skilled agent. That is, a skilled agent is always more productive than an unskilled one when assigned to job $2 .{ }^{8}$ We interpret $H$ as a firm characteristic. Larger $H$ means that human capital is more important at higher hierarchical levels.

Although the principal cannot offer wages contingent on skill acquisition, the principal can commit to a set of non-negative wages $\left(w_{1}, w_{2}\right)$ for the holders of jobs 1 and 2 , respectively. We call $W \equiv w_{2}-w_{1}$ the promotion premium. We describe the compensation contract by a vector $\mathbf{w}=\left(w_{1}, W\right)$ representing a basic reward and a promotion premium. We are interested in the case in which contractual discrimination in promotion decisions is not possible. That is, the principal must offer the same contract, w, to both agents. Because $H>0$, when choosing between two candidates, an unbiased principal always promotes a skilled agent over an unskilled one. As in Prendergast (1993), the principal can effectively commit to rewarding skill acquisition through promotions. In addition, if $l>W$, it is always in the principal's interest to promote one of the agents, even when both agents are unskilled. As $l$ is a free parameter in the model, for simplicity we assume $l>W$. In Section OA.5, we present a variation of the model in which firms sometimes leave job 2 vacant.

As in Section 2, we model the principal's behavior by the probability of choosing Blue given $\omega, s_{b}$ and $s_{r}$. For simplicity, we consider the limiting case in which the principal's subjective information is negligible for the decision: $\omega \rightarrow 0$. In Section OA.3, we present a variation of the model where $\omega$ is non-negligible. There are three cases for the difference in skills: $s_{r}-s_{b}=\Delta s \in$

[^5]$\{-1,0,1\}$. We assume that there is no overt discrimination: if $\Delta s \neq 0$, the principal chooses the skilled agent (Blue if $\Delta s=-1$ and Red if $\Delta s=1$ ). However, if $\Delta s=0$, the principal chooses Blue with probability $0.5+\beta$, where $\beta \in(0,0.5]$ is the principal's subtle bias. Using the notation in Section 2, $\beta=\lim _{\omega \rightarrow 0} \lim _{\Delta s \rightarrow 0} b\left(s_{b}, s_{b}+\Delta s, \omega\right)$, for any $s_{b} \in\{0,1\}$. Thus, given the principal's behavior, the firm's (expected) profit is $\Pi=l+H\left(e_{b}+e_{r}-e_{b} e_{r}\right)-2 w_{1}-W$.

In the context of this model, the principal's behavior can be described by a simple lexicographic heuristic: if $s_{b} \neq s_{r}$, choose the skilled agent; if $s_{b}=s_{r}$, choose Blue with probability $0.5+\beta$. While we assume that ties are exact, this is not a necessary condition for subtle discrimination (see Section 2). ${ }^{9}$ It is also not necessary for the main results; all we need is that observable skills are sufficiently similar so that subtle discrimination is possible. Assuming exact ties is algebraically convenient for solving the firm's maximization problem as described in Subsections 3.4 and 3.5, but otherwise, it is not needed, as we show in Section OA.3.

There are multiple ways to micro-found $\beta$. First, for a taste-based interpretation, suppose that a fraction $2 \beta$ of all firms has a slight preference for blue agents. Agents do not know the type of firm they match with. Thus, in the case of a tie, a blue agent expects to be promoted with probability $2 \beta+(1-2 \beta) \frac{1}{2}=\frac{1}{2}+\beta$. Second, we can give $2 \beta$ an implicit-bias interpretation as the probability that the principal is fooled by its System 1 thinking. Third, we can think of $\frac{1}{2}+\beta$ as beliefs that agents hold about the principal's behavior in the case of a tie. Such beliefs may be correct or incorrect. While, for simplicity, we will assume that these beliefs are correct, most of the results in this paper hold true even when agents' beliefs are incorrect.

### 3.2 Benchmark: First-best Investment Levels

As a benchmark, we consider the problem of an unbiased social planner who maximizes total surplus. Recall that the principal's private subjective information is payoff-irrelevant (i.e., $\omega \rightarrow 0$ ), thus it does not affect the planner's problem. Define (expected) social surplus as $S=\Pi+E\left[u_{b}\right]+$

[^6]$E\left[u_{r}\right]$. The planner's problem is
\[

$$
\begin{equation*}
\max _{\left(e_{b}, e_{r}\right) \in[0,1]^{2}} l+H\left(e_{b}+e_{r}-e_{b} e_{r}\right)-c\left(e_{b}\right)-c\left(e_{r}\right) . \tag{2}
\end{equation*}
$$

\]

A trade-off exists between effort duplication and effort sharing. If both agents invest, some skills might be wasted-a duplication cost. Yet, if only one agent invests, her marginal cost of effort is higher than that of the idle agent. Similar to risk sharing under concave utilities, effort sharing (i.e., marginal cost equalization across agents) is efficient under convex costs. The firstbest choice thus depends on which of these two effects dominates. The following proposition formalizes this intuition (all proposition proofs are in Appendix A):

Proposition 1. The first-best investment levels are either (i) $e_{b}^{F B}=e_{r}^{F B}=\tilde{e}<1$ or (ii) $e_{i}^{F B}>0$ and $e_{-i}^{F B}=0$ for some $i \in\{b, r\}$.

Proposition 1 says that the first-best effort levels can be symmetric or asymmetric. If the benefits from effort sharing are greater than the costs of effort duplication, the social planner chooses the same investment level for both agents (Case (i)). If the duplication costs outweigh the benefits of effort sharing, the social planner asks only one agent to invest in skill acquisition (Case (ii)). As an example, consider $c\left(e_{i}\right)=\frac{k e_{i}^{2}}{2}$. If $H \leq k$, treating both agents equally is socially optimal and the first-best solution is $e_{b}^{F B}=e_{r}^{F B}=\tilde{e}=\frac{H}{H+k}$. If $H>k$, the first-best is a corner solution, $e_{b}^{F B}=1$ and $e_{r}^{F B}=0$ (or $e_{b}^{F B}=0$ and $e_{r}^{F B}=1$ ). That is, it is better to treat agents asymmetrically and make only one of them invest in skill acquisition.

For the rest of the paper, we assume a quadratic cost function, $c\left(e_{i}\right)=\frac{k e_{i}^{2}}{2}$. This assumption allows us to obtain analytical proofs for most results and better explain the economic forces at play. However, it is not crucial for the main results (see Section OA.7).

### 3.3 Equilibrium under Exogenous Compensation Contracts

Here, we describe the agents' investment choices under a fixed contract $\mathbf{w}$. We assume that the contract is individually rational; both Blue and Red accept the contract at Date 0. At Date 1, the
agents simultaneously choose their investment levels. At Date 2, investment outcomes are realized and the principal decides who to promote to the top position. Both agents anticipate that, at Date 2, the principal's decision is biased in favor of Blue.

### 3.3.1 Equilibrium Characterization

We define agent $i$ 's expected utility as:

$$
\begin{equation*}
U_{i}(\mathbf{e}, \mathbf{w}) \equiv w_{1}+W\left[e_{i}\left(1-e_{-i}\right)+\left(\frac{1}{2}+\beta_{i}\right)\left(1-e_{i}-e_{-i}+2 e_{i} e_{-i}\right)\right]-\frac{k e_{i}^{2}}{2}, \tag{3}
\end{equation*}
$$

where $\beta_{b}=-\beta_{r}=\beta$. The first term is the baseline reward, the second term is the promotion premium times the probability of promotion, and the third term is the skill-acquisition cost. The promotion probability is the sum of the probability of agent $i$ being skilled when agent $-i$ is unskilled and the probability of promotion via a tie-breaking decision.

An agent's problem at Date 1 is to maximize his/her expected utility $U_{i}(\mathbf{e}, \mathbf{w})$ by choosing an investment level $e_{i}$ taking the contract, $\mathbf{w}$, and the effort of the other agent, $e_{-i}$, as given. Assuming an interior solution, ${ }^{10}$ the agents' reaction functions are

$$
\begin{equation*}
e_{b}=\frac{W}{k}\left(\frac{1}{2}-\beta+2 \beta e_{r}\right) \text { and } e_{r}=\frac{W}{k}\left(\frac{1}{2}+\beta-2 \beta e_{b}\right) . \tag{4}
\end{equation*}
$$

We define $\sigma \equiv \frac{W}{k}$, i.e., the ratio of the promotion premium to the cost parameter, and call it the premium-cost ratio. Higher $\sigma$ implies a higher net marginal benefit of investment. Intuitively, high $\sigma$ implies that the gain from promotion, $W$, is large relative to the cost of investment, which is proportional to $k$. High $\sigma$ can thus be interpreted as a "high-stakes" career path, i.e., there is much to gain from investing in skill acquisition. In contrast, if $\sigma$ is low, agents benefit little from investing; the agents are on a low-stakes career path. Thus, we also informally refer to $\sigma$ as the "stake" of a career path.

In the baseline case with no subtle bias $(\beta=0)$, the reaction functions in (4) are flat, implying

[^7]that $e_{b}^{*}=e_{r}^{*}=\frac{\sigma}{2}$ is the dominant strategy. To understand the intuition for this result, suppose agent $i$ increases her investment by $\varepsilon$. If the other agent is skilled, agent $i$ 's promotion probability increases by $\frac{\varepsilon}{2}$ because she can be promoted only when she is skilled, in which case the winner is chosen with equal probabilities. If the other agent is unskilled, agent $i$ increases her promotion probability by $\varepsilon-\frac{\varepsilon}{2}=\frac{\varepsilon}{2}$, because if she is skilled, she is promoted with certainty and if she is unskilled, both agents are equally likely to be promoted. In sum, agent $i$ 's promotion probability always increases by half of her investment increase, regardless of whether the other agent is skilled or unskilled. ${ }^{11}$

The introduction of a bias in favor of Blue breaks this symmetry. Now, if Blue increases his effort by $\varepsilon$, his promotion probability increases by $\varepsilon\left(\frac{1}{2}+\beta\right)$ in the state where Red is skilled, while in the state where Red is unskilled, his promotion probability increases by less: $\varepsilon\left(\frac{1}{2}-\beta\right)$. Thus, the state matters for the investment decision: Blue's marginal benefit of investing is larger when Red is more likely to be skilled. That is, Blue's reaction function is positively sloped. For similar reasons, Red's reaction function is negatively sloped. Intuitively, ties are more valuable to Blue than they are to Red. Thus, Blue wants to imitate Red's behavior, which causes Blue's reaction function to slope upwards. In contrast, Red adopts the opposite strategy in an attempt to avoid ties. The following proposition characterizes the equilibrium investment choices.

Proposition 2. A unique equilibrium exists. For any $\beta \in[0,0.5]$, there exists $\bar{\sigma}(\beta)>1$ (with $\bar{\sigma}(0.5)=\infty)$ such that, if $\sigma \leq \bar{\sigma}(\beta)$, the equilibrium is interior and the investment levels are given by:

$$
\begin{align*}
& e_{b}^{*}=\frac{\sigma(0.5-\beta)+2 \beta \sigma^{2}(0.5+\beta)}{1+4 \beta^{2} \sigma^{2}}  \tag{5}\\
& e_{r}^{*}=\frac{\sigma(0.5+\beta)-2 \beta \sigma^{2}(0.5-\beta)}{1+4 \beta^{2} \sigma^{2}} \tag{6}
\end{align*}
$$

If $\sigma>\bar{\sigma}(\beta), e_{b}^{*}=1$ and $e_{r}^{*}=\min \left\{\frac{\sigma(1-2 \beta)}{2}, 1\right\} .{ }^{12}$

[^8]
### 3.3.2 Discouragement versus Overcompensation

Figure 1 shows the equilibrium investment levels as a function of the premium-cost ratio, $\sigma$, for two levels of the subtle bias. The figure shows that for low values of $\sigma$, Red invests more than Blue, while for high values of $\sigma$, it is Blue who invests more. The following corollary formalizes the


Figure 1: Equilibrium investments, $e_{b}^{*}$ and $e_{r}^{*}$, as functions of the premium-cost ratio, $\sigma$, for two levels of subtle bias, $\beta_{1}=0.1$ and $\beta_{2}=0.4$.
comparative statics illustrated in Figure 1 (all corollary proofs are in Section OA.1). For simplicity of exposition, from now on we assume that the equilibrium is interior.

Corollary 1. When stakes are low, Red invests more than Blue. When stakes are high, Blue invests more than Red. Formally, $e_{r}^{*} \geq e_{b}^{*}$ if and only if $\sigma \leq 1$.

Red's investment decision is shaped by two opposing forces. On the one hand, Red wants to invest heavily in skills to try to separate herself from Blue. We call this force the overcompensation effect. Overcompensation may occur because the red agent knows she is held to "higher standards:" unless she is clearly more qualified than Blue, she is viewed less favorably. On the other hand, Red is discouraged from investing because her chances of promotion are slim even if she acquires the skill. We call this force the discouragement effect. ${ }^{13}$ Parameter $\sigma$ determines which effect dominates in equilibrium. If the stakes are low, Blue exerts low effort. Thus, Red is willing to overcompensate

[^9]by investing more, both because the probability of separation is high and because the marginal cost of investing is low under a convex cost function. As the stakes increase, Blue chooses higher levels of investment, discouraging Red from investing. At high investment levels, the probability of separation is low while the marginal cost of investing is high.

Remark 1. If $l<0$, job 2 may stay vacant in some cases, and both reaction functions slope downwards (see Section OA.6). In this case, the discouragement effect always dominates the overcompensation effect, and Red invests less than Blue in equilibrium. Focusing on the $l>0$ case makes the model richer because either effect may dominate in equilibrium.

Remark 2. The interaction between the overcompensation and discouragement effects is robust to situations in which Blue and Red have different beliefs about $\beta$. For example, if Red believes that there is subtle discrimination $(\beta>0)$, but Blue believes that $\beta$ is zero, we have $e_{b}^{*}=\frac{\sigma}{2}$ and $e_{r}^{*}=\sigma\left(\frac{1}{2}+\beta(1-\sigma)\right)$, implying that Corollary 1 holds.

Remark 3. A potential consequence of the discouragement effect is that a principal who is unaware of his bias (and the strategic interaction it creates between the two agents) might interpret Red's behavior as a lack of interest in high-paying positions. In other words, he might incorrectly "learn" that red and blue agents have different preferences with respect to earned income. Such learning might further reinforce the principal's subtle bias or even result in an explicit bias in favor of blue agents. ${ }^{14}$

### 3.3.3 Subtle Discrimination versus Overt Discrimination

To understand how the model's predictions relate to the type of discrimination, we consider how these predictions would change in the presence of overt discrimination. We define the overt bias as $\delta=\lim _{\omega \rightarrow 0} b(0,1, \omega)$. If $\delta>0$, the reaction functions become

$$
\begin{equation*}
e_{b}=\sigma\left[\frac{1}{2}-\beta+(2 \beta-\delta) e_{r}\right] \text { and } e_{r}=\sigma\left[\frac{1}{2}+\beta-\delta-(2 \beta-\delta) e_{b}\right] \tag{7}
\end{equation*}
$$

[^10]Thus far, we have imposed no structure on $\beta$ and $\delta$ other than $\delta \leq 0.5+\beta$, which follows from $P\left(s_{b}, s_{r}, \omega\right)$ being decreasing in $s_{r}$ and increasing in $s_{b}$. Without further structure, we do not know whether the reaction functions in (7) have positive or negative slopes. To impose further (but minimal) structure, we can think of overt discrimination as a probability over two states of the world: in the first state, which happens with probability $\delta$, the principal has a large bias towards Blue and, thus, chooses Blue with probability one, while in the second state, which happens with probability $1-\delta$, the principal's bias is small, thus he still chooses Red if $s_{b}=0$ and $s_{r}=1$. While in State 2 discrimination does not occur when $s_{b}=0$ and $s_{r}=1$, it may still occur when $\Delta s=0$ (both agents have the same skill) because even a small bias might affect decisions in this case. Formally, this implies that $\delta$ imposes a lower bound on $\beta: \frac{1}{2}+\beta \geq \delta+(1-\delta) \frac{1}{2}$, which implies $2 \beta \geq \delta$.

Let $\varepsilon \equiv \beta-\frac{\delta}{2} \geq 0$ denote the excess subtle bias. If $\varepsilon=0$ (no excess subtle bias), the reaction functions in (7) are again flat, implying $e_{b}=e_{r}=\frac{\sigma(1-\delta)}{2}$ (for $\delta<1$ ). That is, if subtle discrimination is fully "explained" by an overt bias, both agents choose the same investment levels in equilibrium. Asymmetric investment levels occur only when subtle discrimination is stronger than overt discrimination. In other words, overt discrimination moderates the effect of subtle discrimination.

We now generalize Corollary 1 for the case in which overt discrimination is present:

Corollary 2. If $\delta \geq 0, e_{r}^{*} \geq e_{b}^{*}$ if and only if $\sigma \leq \frac{1}{1-\delta}$.
Corollary 2 shows that overt discrimination makes it less likely that Red invests more than Blue. Intuitively, under overt discrimination, Red benefits less from separating herself from Blue because, even when Red is more skilled than Blue, she is still passed over for promotion with positive probability. That is, the overcompensation effect is weaker when the principal also overtly discriminates. These results show that subtle discrimination has unique implications. In the context of our model, subtle discrimination creates incentives for separation, while overt discrimination does not. Only in the presence of excess subtle discrimination can the overcompensation effect dominate the discouragement effect. Therefore, our model is particularly relevant when overt
discrimination is mild or nonexistent, but subtle discrimination remains.
In what follows, we consider the case of "pure" subtle discrimination, i.e., $\delta=0$. All results are qualitatively unchanged if we interpret $\beta$ as excess subtle bias.

### 3.3.4 Stakes, Investment in Skills, and the Promotion Gap

The next corollary presents further comparative statics results.

Corollary 3. For $\sigma \leq \bar{\sigma}(\beta)$ (i.e., the equilibrium is interior), we have that

1. $e_{b}^{*}$ is strictly increasing in $\sigma$;
2. There exists $\widehat{\sigma}(\beta) \leq \bar{\sigma}(\beta)$ such that $e_{r}^{*}$ increases with $\sigma$ for $\sigma \leq \widehat{\sigma}(\beta)$ and decreases with $\sigma$ for $\sigma>\widehat{\sigma}(\beta)$.
3. $\widehat{\sigma}(\beta)$ is strictly decreasing in $\beta$.

Part 1 shows that Blue's investment in skill acquisition is increasing in the premium-cost ratio. Part 2 shows that Red's investment does not always increase with $\sigma$. If the stakes are sufficiently high $(\sigma>\hat{\sigma}(\beta))$, the discouragement effect dominates and Red's investment declines with the premium-cost ratio (for this to happen, the subtle bias needs to be sufficiently strong). Part 3 shows that when the bias is stronger, the discouragement effect is also stronger, implying a lower premium-cost ratio at which Red's investment declines with $\sigma$.

Let $p_{i}$ denote agent $i$ 's promotion probability, $i \in\{b, r\}$. The promotion gap between blue and red agents is

$$
\begin{equation*}
\Delta p \equiv p_{b}-p_{r}=e_{b}-e_{r}+\left[e_{b} e_{r}+\left(1-e_{b}\right)\left(1-e_{r}\right)\right] 2 \beta . \tag{8}
\end{equation*}
$$

The promotion gap has two terms. The first term, $e_{b}-e_{r}$, is the difference in the probabilities of skill acquisition. Given our broad interpretation of skill, we call this difference the achievement gap. The second term is the difference in promotion probabilities between Blue and Red that arises as a direct consequence of the subtle bias. It is the promotion gap conditional on a tie times the probability of a tie. We call this term the favoritism gap. The subtle bias affects both
the achievement gap and the favoritism gap. The next proposition shows how the equilibrium promotion gap varies with the premium-cost ratio, $\sigma$.

Proposition 3. For each $\beta \in(0,0.5]$, there exists a unique premium-cost ratio $\widetilde{\sigma}(\beta)$ such that the promotion gap decreases in $\sigma$ for $\sigma<\widetilde{\sigma}(\beta)$ and increases in $\sigma$ for $\sigma \in(\widetilde{\sigma}(\beta), \bar{\sigma}(\beta))$.

Figure 2 illustrates how the promotion gap changes with the premium-cost ratio, $\sigma$. The promotion gap initially decreases with $\sigma$ and then increases with $\sigma$. For large values of the premium-cost ratio, even a small subtle bias can be significantly amplified through the strategic interactions between the agents. Also, in high-stakes careers, the contribution of the achievement gap to the promotion gap is greater than that of the favoritism gap. That is, differences in "observable" achievements (human capital, performance, experience, effort, etc.) explain most of the promotion gap. In other words, because ties occur less frequently as the stakes increase, the principal is less likely to make biased promotion decisions. In such scenarios, we would expect to find little direct evidence of discrimination. Nevertheless, promotion gaps are large in high-stakes situations.


Figure 2: Equilibrium promotion gap, $\Delta p^{*}$, as a function of the premium-cost ratio, $\sigma$, for two levels of subtle bias, $\beta_{1}=0.1$ and $\beta_{2}=0.4$.

### 3.4 Optimal Compensation Contracts

We now allow the principal to design the compensation contract. As standard in principal-agent problems, we assume that the firm is a monopsonist in the labor market; that is, the firm has all
the bargaining power. The principal is not allowed to discriminate through contracts explicitly; he must offer the same contract $\mathbf{w}=\left(w_{1}, W\right)$ to both agents. Agents are assumed to be penniless; wages must be non-negative: $w_{1} \geq 0$ and $w_{1}+W \geq 0$.

To remain in a fully rational world, we assume that the principal knows that the agents behave as if promotions are subject to subtle bias $\beta$. One interpretation is that the principal is aware of his own bias. Under this interpretation, the subtle bias may create a dynamic inconsistency problem: the principal could be (in some cases) better off by committing not to discriminate, but there is no commitment technology available. In the language of O'Donoghue and Rabin (1999), the principal is a "sophisticate," i.e., someone who understands that they will subtly discriminate and, therefore, can correctly predict their future behavior. A second - and perhaps more empirically relevant - interpretation is that promotion decisions are made by a biased third party (e.g., a direct supervisor), and the principal designs the contract taking into account the third party's bias (see Prendergast and Topel (1996) for a model along these lines).

Agents' outside utilities are normalized to zero. To avoid corner solutions, we assume that the firm pays a fixed entry cost to operate; to save on notation, we assume that this cost is $l+\varepsilon$, with $\varepsilon$ arbitrarily small. This assumption implies that the firm chooses to operate if and only if the expected profit after entry is strictly greater than $l$. The principal is risk-neutral and derives no utility from discrimination. His profit-maximization problem (after entry, i.e., gross of entry costs) is as follows:

$$
\begin{equation*}
\max _{w_{1} \geq 0, w_{1}+W \geq 0} l+H\left(e_{b}+e_{r}-e_{b} e_{r}\right)-2 w_{1}-W, \tag{9}
\end{equation*}
$$

subject to

$$
\begin{equation*}
e_{i}=\arg \max _{e \in[0,1]} e W\left[\left(\frac{1}{2}-\beta_{i}\right)+2 \beta_{i} e_{-i}\right]-\frac{k e^{2}}{2}, \text { for } i \in\{b, r\}, \tag{10}
\end{equation*}
$$

where $\beta_{b}=-\beta_{r}=\beta$. The principal faces two incentive compatibility (IC) constraints in (10). The agents' participation constraints are not binding because $w_{1} \geq 0$ and $w_{1}+W \geq 0$ imply that agent $i$ can guarantee a non-negative payoff by choosing $e_{i}=0$. Because $w_{1}$ does not affect the IC constraints, the principal optimally sets $w_{1}=0$. If the principal chooses $w_{1}=W=0$, the agents
exert no effort, and the post-entry profit is $l$. In such a case, the firm's profit from entering the market is $l-l-\varepsilon<0$. Thus, we use $w_{1}=W=0$ to denote the case in which the firm does not operate.

For any given $k$, choosing the wage upon promotion, $W$, is equivalent to choosing the stake, $\sigma$. Henceforth, for convenience and clarity, we view the principal's task as selecting $\sigma$. Proposition 2 implies that $e_{b}=1$ if $\sigma>\bar{\sigma}(\beta)$, thus increasing $\sigma$ beyond $\bar{\sigma}(\beta)$ has no impact on revenue. That is, in an optimal contract, $\sigma \leq \bar{\sigma}(\beta)$. With these observations, the principal's problem can be written as:

$$
\begin{equation*}
\Pi(k, \beta, \theta)=\max _{\sigma \in[0, \bar{\sigma}(\beta)]} k \theta\left(e_{b}+e_{r}-e_{b} e_{r}\right)-k \sigma \tag{11}
\end{equation*}
$$

subject to (5) and (6), where $\theta \equiv \frac{H}{k}$ is the productivity-cost ratio and $\Pi(k, \beta, \theta)$ is the optimal expected profit net of entry costs (as $\varepsilon \rightarrow 0$ ). Parameter $\theta$ can also be interpreted as a measure of the relative importance of human capital at higher hierarchical levels. Thus, for interpretation, we call firms with high $\theta$ human-capital-intensive firms.

We first solve a baseline case in which $\beta=0$.

Proposition 4. If the principal is unbiased $(\beta=0)$, the firm operates if and only if $\theta>1$ and the optimal stake, $\sigma^{*}=\frac{2(\theta-1)}{\theta}$, uniquely implements investment levels $e_{b}^{*}=e_{r}^{*}=\frac{\theta-1}{\theta}$.

When there is no bias, both agents choose the same investment level in equilibrium. The firm operates only when $\theta>1$, i.e., the productivity gain for the principal is high relative to the marginal cost of investment for the agents. That is, firms with low productivity-cost ratios prefer to shut down. From a social welfare perspective, all firms should operate because when $e_{b}=e_{r}=0$, the marginal cost of investing is zero, while the marginal social benefit of investing is positive and equal to $H>0$. Thus, when $\theta \leq 1$, the firm inefficiently stays out of business. Such inefficiency occurs because the non-negative wage constraint prevents the firm from extracting all the surplus from the agents.

In the general case $(\beta \geq 0)$, an analytical solution is not universally attainable. However, it is possible to prove the existence and uniqueness of the optimal contract:

Proposition 5. For every set of parameters $(k>0, \beta \in[0,0.5], \theta>0)$, there exists a unique solution $\sigma(k, \beta, \theta)$ to the principal's problem (if the firm chooses not to operate, we set $\sigma=0$ ).

Without loss of generality, from now on, we set $k=1$. Let $\sigma(\beta, \theta)$ denote the optimal stake. The next result describes the properties of the optimal contract and how it changes with the productivity-cost ratio, $\theta$.

Proposition 6. For every $\beta \in[0,0.5]$, there exist values $\underline{\theta}(\beta)<\bar{\theta}(\beta)$ such that:

1. If $\theta \leq \underline{\theta}(\beta)$, the optimal stake is $\sigma(\beta, \theta)=0$ (i.e., the firm does not operate). If $\theta \geq \bar{\theta}(\beta)$, the optimal stake is $\sigma(\beta, \theta)=\bar{\sigma}(\beta)$.
2. The optimal stake, $\sigma(\beta, \theta)$, is strictly increasing in $\theta \in[\underline{\theta}(\beta), \bar{\theta}(\beta)]$.
3. The firm's profit is strictly increasing in $\theta \geq \underline{\theta}(\beta)$.

Part 2 of Proposition 6 implies that human-capital-intensive firms (high- $\theta$ firms) offer career paths involving higher stakes. Because the optimal stake is increasing in $\theta$, all the comparative statics in the previous subsection are unchanged once we replace $\sigma$ with $\theta$. In particular, if we define $\widetilde{\theta}(\beta)$ as the value of $\theta$ such that the optimal stake is $\sigma(\beta, \widetilde{\theta}(\beta))=1$, we again have that Red invests more than Blue when stakes are low $(\theta<\widetilde{\theta}(\beta))$ and Blue invests more than Red when stakes are high $(\theta>\widetilde{\theta}(\beta))$. Finally, Part 3 implies that high- $\theta$ firms are more profitable. Thus, we can also use $\theta$ as a proxy for firm profitability or productivity. Panels (a) and (b) of Figure 3 illustrate Proposition 6 (for $\beta=0.4$ ), while panel (c) shows that similar to Figure 2, the equilibrium promotion gap is U -shaped in the productivity-cost ratio, $\theta$.

### 3.5 Optimal Anti-Discrimination Policies

Does subtle discrimination benefit or harm firms? To see how subtle discrimination affects profits, we now consider the problem of a principal who can choose both the compensation contract and the firm's own subtle bias. We have in mind a situation in which the firm chooses an optimal anti-discrimination policy. For example, the firm can set up processes that lead to the selection of


Figure 3: Optimal premium-cost ratio, equilibrium profit, and promotion gap, as functions of the productivity-cost ratio, $\theta$ for a given level of subtle bias ( $\beta=0.4$ ).
supervisors with high or low subtle biases. Similarly, the firm can invest in a corporate culture that is either friendly or hostile to diversity goals. Firms can become conservative by adopting policies associated with high $\beta$. Similarly, firms can become progressive by adopting policies associated with low $\beta$.

Suppose the principal can select policies associated with bias $\beta$ at no cost (another interpretation is that market forces drive firms with suboptimal biases out of the market). Which $\beta$ would the principal choose? The principal's problem is

$$
\begin{equation*}
\Pi(\theta)=\max _{(\sigma, \beta) \in[0, \bar{\sigma}(\beta)] \times[0,0.5]} \theta\left(e_{b}+e_{r}-e_{b} e_{r}\right)-\sigma \tag{12}
\end{equation*}
$$

subject to (5) and (6). Let $\beta(\theta)$ denote the profit-maximizing subtle bias and $\sigma(\theta)$ the corre-
sponding optimal stake. Define $\bar{\theta}$ as the lowest value of $\theta$ such that $\sigma(\theta)=\bar{\sigma}(\beta(\theta))$. That is, the optimal stake is strictly interior if and only if $\theta \leq \bar{\theta}$. From now on, we focus on strictly interior solutions for $\sigma$.

One might think that firms would obviously choose to have no bias, i.e., $\beta=0$. Indeed, in the unconstrained first-best (assuming both workers need to be hired), both workers invest the same amount. However, the unconstrained first-best is typically not achievable because the non-negative wage constraint prevents the firm from extracting all the surplus from the workers. While nonnegative constraints are just a modeling feature here, in the real world workers face borrowing constraints. Thus, workers typically need to earn a minimum (non-zero) wage. This constraint is more likely to be binding in low-stakes situations. Due to this friction, it is no longer obvious that the firm prefers to treat all agents equally, i.e., to set $\beta=0$. The contest literature shows that asymmetric contests may be optimal even when agents are identical (see Kawamura and de Barreda (2014) and Drugov and Ryvkin (2017)). However, most papers in this literature do not explicitly model the frictions that may lead to the optimality of biased contests. In particular, no paper in this literature distinguishes between subtle and overt biases. The next proposition shows how a subtle bias might be optimal when firms cannot extract all the surplus through low initial wages:

Proposition 7. There exists $\theta^{\prime}<\bar{\theta}$ such that

$$
\beta(\theta)=\left\{\begin{array}{cc}
0.5 & \text { if } \theta \in\left(0, \theta^{\prime}\right]  \tag{13}\\
0 & \text { if } \theta \in\left[\theta^{\prime}, \bar{\theta}\right]
\end{array}\right.
$$

Furthermore, $\sigma(\theta)<1$ if $\theta \in\left(0, \theta^{\prime}\right]$ and $\sigma(\theta)>1$ if $\theta \in\left[\theta^{\prime}, \bar{\theta}\right]$.

This proposition shows that if the principal could optimally choose his subtle bias (or, equivalently, a supervisor with a given bias) at no cost, he would always choose a corner solution for the bias: either no bias or the maximum bias. This choice is determined by the productivity-cost ratio, $\theta$. Figure 4 illustrates the optimal stake, profit and the resulting promotion gap as functions of $\theta$. For less productive firms, i.e., firms with low $\theta$, profits increase with subtle discrimination.

Thus, firm profit is maximized at $\beta^{*}=0.5$. Such firms also optimally offer low-stakes careers, $\sigma(\theta)<1$. Intuitively, subtle discrimination is profitable for firms that offer low-stakes careers because the overcompensation effect improves the performance of discriminated agents. Thus, in less productive (or less human-capital-intensive) sectors, firms perform better when they discriminate.


Figure 4: Optimal premium-cost ratio, equilibrium profit, and promotion gap, as functions of the productivity-cost ratio, $\theta$.

In contrast, high- $\theta$ firms greatly profit from promoting skilled agents and are therefore willing to offer high stakes to incentivize agents to invest in their skills. However, under high stakes, the discouragement effect hinders the investment of discriminated agents. Thus, to counter the discouragement effect, high- $\theta$ firms prefer to choose robust anti-discrimination policies. That is, in high- $\theta$ firms, the maximal profit is achieved with zero subtle bias. This result implies that in sectors with high $\theta$, discriminating firms are less profitable than non-discriminating firms and, thus, they are more likely to be driven out by competition.

When firms optimally choose their biases, high- $\theta$ firms do not have a promotion gap (see Figure $4 \mathrm{c})$. That is, such firms have more diversity at the top. In contrast, low- $\theta$ firms have positive promotion gaps. Thus, our model predicts a particular type of firm polarization, in which high- and lowproductivity firms choose different policies with respect to discrimination and diversity. Note that our model predicts a large promotion gap for high-productivity firms conditional on a given bias. When firms can choose their biases, high-productivity firms have small promotion gaps.

The results concerning firm policies can be summarized as follows. High-productivity firms prefer to promote a discrimination-free work environment. They strive to be perceived as "progressive" and "activist." If successful, they would have more diversity at the top (i.e., a smaller promotion gap). These firms also offer careers with higher stakes and are likely to be large, profitable, and human capital-intensive. In contrast, low-productivity firms do not take actions to counter subtle discrimination. They do not mind being perceived as "conservative" and have less diversity at the top. They offer careers with low stakes and are smaller, less profitable, and less human capital-intensive than "progressive" firms. Note that polarization implies that two firms with productivity-cost ratios $\theta^{\prime}+\eta$ and $\theta^{\prime}-\eta$ adopt very different anti-discrimination policies even if their differences in productivity are negligible (i.e., as $\eta \rightarrow 0$ ).

As we show in Section OA.9, a larger bias can increase or decrease social welfare. Perhaps surprisingly, for low- $\theta$ firms, a larger bias might be Pareto improving. In such firms, even red agents can be better off under a larger $\beta$ because the firm offers a higher promotion premium in an attempt to exploit the overcompensation effect.

## 4 Related Literature

Lazear and Rosen (1990) present a model in which men have higher promotion rates than equally qualified women because women have better non-work opportunities and are thus more likely to quit after being promoted. They claim that the evidence that the gender wage gap is mainly caused by gender promotion gaps is "difficult to incorporate into the main economic theory of
discrimination based on taste factors alone" (p. S107). In contrast, our model shows that subtle discrimination can cause large promotion gaps even when there are no differences in pay across groups within a job.

Our setup is similar in spirit, though not in detail, to that of Prendergast (1993), who proposes a model of promotions in which the firm cannot contractually commit to compensating workers for acquiring firm-specific human capital. Our model differs in two significant aspects: i) promotions are competitive, i.e., candidates compete for a limited number of positions; ii) the principal is subtly biased in favor of candidates from a particular group.

Our model relates to the literature on favoritism and other biases in subjective performance evaluations and their consequences for selection and promotion decisions (Prendergast and Topel (1996); MacLeod (2003); Friebel and Raith (2004); Hoffman et al. (2018); Frederiksen et al. (2020); Letina et al. (2020); Frankel (2021); Pagano and Picariello (2022)). In these models, favoritism and other biases have ex-post payoff consequences for the decision-maker. In contrast, in our model, favoritism matters only because it affects ex-ante incentives; it has no ex-post (i.e., after investment) consequences.

More broadly, our study is related to the theoretical literature on discrimination (see Fang and Moro (2011), Lang and Lehmann (2012), and Onuchic (2023) for reviews). In their seminal work on affirmative action, Coate and Loury (1993) show that negative stereotypes can be selffulfilling as discriminated agents may not undertake investments that make them more productive. Similarly, in our model, discrimination may discourage some agents from investing. However, because workers compete for the same position, their investment decisions are interdependent. Such strategic considerations may further discourage investment or, instead, provide discriminated agents with stronger incentives to invest. Thus, differently from Coate and Loury (1993), in our model, the unfavored group may invest more than the favored group. This result is obtained only when discrimination is subtle. In addition, the strategic interactions between agents imply a unique equilibrium, which is rare in the literature on self-fulfilling discrimination. ${ }^{15}$ Building upon Coate

[^11]and Loury (1993), Fryer Jr. (2007) presents a statistical discrimination model with two sequential stages: hiring and promotion. In his model, employer beliefs may "flip" between stages because, once hired, the disadvantaged worker is expected to be more qualified than the favored worker. Our model has a similar flavor in that promoted red agents are more qualified on average. However, because qualifications are observable, subtle biases do not flip.

In our model, there are peer effects: agents impose externalities on each other. In this sense, our model is related to those by Mailath et al. (2000), Moro and Norman (2004) and Chaudhuri and Sethi (2008) who study labor markets where workers from one group impose externalities on another group. In these models, asymmetric equilibria exist in which agents with identical ex-ante qualifications receive different wages. In contrast, in our model, wages are not conditional on agents' labels, and therefore discrimination cannot be verified ex-post. In a recent study, Onuchic and Ray (2023) study a setting where individuals can collaborate in pairs and show that small biases in public attribution of credit can be amplified by the strategic responses of individuals. In our model, subtle biases can be either amplified or attenuated depending on the stakes.

Unlike theories of discrimination based on differential screening abilities (Cornell and Welch (1996); Fershtman and Pavan (2021)), our model assumes that the principal knows each candidate's type. While we can still interpret subtle discrimination as a form of incorrect or exaggerated beliefs, as in Bordalo et al. (2016), it can also be seen as a limiting case of taste-based discrimination when the taste parameter is arbitrarily small.

Our paper is also related to a strand of the discrimination literature that focuses on bias amplification. Lang et al. (2005) show that in markets where firms post wages, weak discriminatory preferences can cause large wage differentials. Bartoš et al. (2016) show how "attention discrimination" can amplify animus and prior beliefs about group quality. Davies et al. (2021) demonstrate that an arbitrary small bias towards one candidate can have large consequences when the principal exerts effort to learn about candidates' abilities. Siniscalchi and Veronesi (2021) present a model in which mild population heterogeneity and self-image bias can lead to persistent differences between groups. In our model the source of bias amplification is the competitive nature of promotion
tournaments and agents can both magnify and attenuate small biases through their actions. ${ }^{16}$
Additionally, our paper is related to a small theoretical literature on biased contests (Kawamura and de Barreda (2014); Pérez-Castrillo and Wettstein (2016)). Drugov and Ryvkin (2017) show that under certain conditions, biased contests can be optimal from the organizer's point of view (e.g., total effort maximization) even when contestants are symmetric. In that vein, Nava and Prummer (2022) present a model in which the principal can directly affect the contestants' valuations of the prize (promotion) through work culture. Our paper differs from this literature in many respects, particularly in our focus on subtle discrimination, its empirical implications, and its consequences for different types of firms.

Finally, our notion of subtle bias is related to Cunningham and de Quidt's (2022) concept of implicit preferences, defined as preferences toward a trait (like gender or race) that have a stronger effect when mixed with other characteristics. They focus on developing a methodology for identifying such preferences from choices. Our main focus is on how subtle biases (which encompass both their implicit and explicit categories) impact human capital accumulation and firm outcomes.

## 5 Empirical Implications and Conclusions

Most cases of discrimination we witness in day-to-day life are subtle. Subtle discrimination leaves no trace and is subject to plausible deniability. Although subtle discrimination may harm those at the receiving end, it may not have many immediate consequences for the perpetrating parties. Our leading example of subtle discrimination is the use of biased tie-breaking rules in promotion contests.

Our model generates several novel predictions. The model shows that unfavored agents are discouraged to invest in human capital when promotion stakes are high. While it is not always clear how to measure "promotion stakes," the gain from promotion is likely related to the importance

[^12]of human capital for performing a task. For example, promotion benefits are widely perceived to be high in professional services careers, such as consulting, law, and finance. Azmat et al. (2020) find that the differences in promotion rates between men and women in law firms are explained by men working more hours (i.e., exerting more effort) than women in entry-level positions. Such evidence is consistent with a discouragement effect in high-stakes careers. In contrast, Benson et al. (2021) find that women on management-track careers in retail have better pre-promotion performance than men. This finding is consistent with an overcompensation effect that dominates in low-stakes situations. Our model also predicts that, in competitive contexts, observable skills are more valuable to unfavored groups. Consistent with this prediction, Niessen-Ruenzi and Zimmerer (2023) find that women's career outcomes are more sensitive to observable skills than those of men.

There are several ways in which one can test for subtle discrimination in competitive situations. One is to identify a direct shock to the bias. According to our model, an ex-post, unanticipated preference shock would change promotion gaps but would have no impact on firm performance in the short run. As an example of this approach, Ronchi and Smith (2021) find evidence that an exogenous shock to male managers' gender attitudes - the birth of a daughter as opposed to a son - increases managers' propensity to hire female workers. They also find that the shock has no effect on firm performance, which is explained by managers replacing men with women with comparable qualifications, experience, and earnings. Overall, the evidence is consistent with subtle discrimination affecting gender gaps but not profits in the short run.

Our model also explains why some firms invest in building a "progressive" corporate culture while others are content to maintain a "conservative" image. Subtle discrimination is detrimental to high-productivity firms because discriminated workers are discouraged from investing in valuable skills. Thus, such firms prefer to foster equality as a means to incentivize a diverse workforce. In contrast, low-productivity firms benefit from holding discriminated workers to higher standards, as these employees overcompensate by working harder. Consistent with our predictions, Edmans et al. (2023) find that employees' perception of diversity, equity and inclusion is stronger in growing, high-valuation, and financially strong firms. Similarly, a robust empirical finding is that, in the
cross-section, large and high-performing firms have more women on their boards (see, e.g., Adams and Ferreira (2009)). Consistent with these cross-sectional correlations, Gao et al. (2023) find that firms experiencing a shock that lowers their cost of capital reduce their racial promotion gaps for mid- and high-skill workers. We are unaware of theoretical work explaining this body of evidence. Subtle discrimination can explain these findings.

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## Appendix

## A Proofs

Proof of Proposition 1. Suppose first that both agents undertake strictly positive investments in skill acquisition in the first-best solution. The first-order conditions (FOCs) for an interior solution are $H\left(1-e_{j}\right)-c^{\prime}\left(e_{i}\right)=0$, for $i \neq j \in\{b, r\}$. Under $c^{\prime \prime}\left(e_{i}\right)>0$, an interior solution must be unique, which implies that the solution is symmetric and given by $\tilde{e}$, where $\tilde{e}=1-\frac{c^{\prime}(\tilde{e})}{H}$. Note that $\tilde{e}$ is well defined as long as $H>c^{\prime}(0)$. We extend the definition of $\tilde{e}$ so that $\tilde{e}=0$ if $H \leq c^{\prime}(0)$. We can then calculate the surplus associated with $\tilde{e}: \tilde{S} \equiv H \tilde{e}(2-\tilde{e})-2 c(\tilde{e})$.

Consider now the case in which only one agent, say $b$, is requested to exert effort. If $H>c^{\prime}(0)$, the optimal investment is given by $\hat{e}_{b}=\min \left\{c^{\prime-1}(H), 1\right\}$. If $H \leq c^{\prime}(0)$, we set $\hat{e}_{b}=0$. The surplus associated with $\hat{e}_{b}$ is $\hat{S} \equiv H \hat{e}_{b}-c\left(\hat{e}_{b}\right)$.

The first-best investment levels can take one of two forms. If $\tilde{S} \geq \hat{S}$, the gains from sharing effort are greater than the losses from effort duplication, in which case we have $e_{b}^{F B}=e_{r}^{F B}=\tilde{e}$. If, instead, $\tilde{S}<\hat{S}$, effort duplication is too costly, thus the first-best solution is $e_{b}^{F B}=\hat{e}_{b}$ and $e_{r}^{F B}=$ 0 .

Proof of Proposition 2. Equations (5) and (6) represent the unique solution to the system of equations given by (4). From (5), we find that $e_{b}^{*} \leq 1$ requires $f_{b}(\sigma)=\beta(2 \beta-1) \sigma^{2}-(0.5-\beta) \sigma+$ $1 \geq 0$. Function $f_{b}$ is strictly concave and has a unique positive root, $\bar{\sigma}(\beta) \equiv \frac{\beta-0.5+\sqrt{\frac{1}{4}+3 \beta-7 \beta^{2}}}{2 \beta(1-2 \beta)} \geq$ 0 , for all $\beta \in(0,0.5)$. Thus, $e_{b}^{*} \leq 1$ if and only if $\sigma \leq \bar{\sigma}(\beta)$.

To show that $\bar{\sigma}(\beta)>1$, note that

$$
\beta-0.5+\sqrt{\frac{1}{4}+3 \beta-7 \beta^{2}}=\beta-0.5+\sqrt{(\beta-0.5)^{2}+4 \beta(1-2 \beta)}=-a+\sqrt{a^{2}+2 b}
$$

where $a=0.5-\beta>0$ and $b=2 \beta(1-2 \beta)>0$. Then, $-a+\sqrt{a^{2}+2 b}>b \Longleftrightarrow a^{2}+2 b>$ $a^{2}+2 a b+b^{2} \Longleftrightarrow 2>b+2 a \Longleftrightarrow 2>2 \beta-4 \beta^{2}+1-2 \beta \Longleftrightarrow 1>-4 \beta^{2}$.

Similarly, $e_{r}^{*} \leq 1$ requires $f_{r}(\sigma)=\beta(2 \beta+1) \sigma^{2}-(0.5+\beta) \sigma+1 \geq 0$. Function $f_{r}$ is strictly
convex. If $f_{r}$ has no real root, then trivially $e_{r}^{*}<1$ for any value of $\sigma$. A pair of real roots exists when $\beta \in\left(0, \frac{1}{14}\right]$. In this case, the smallest real root is: $\frac{0.5+\beta-\sqrt{\frac{1}{4}-3 \beta-7 \beta^{2}}}{2 \beta(2 \beta+1)}$, which is greater than 1 ; this is shown by setting $a=0.5+\beta, b=2 \beta(1+2 \beta)$ and verifying that $a-\sqrt{a^{2}-2 b}>b$.

Note also that $f_{r}(\sigma)-f_{b}(\sigma)=2 \beta \sigma(\sigma-1)$, which is positive if $\sigma>1$. Thus, if $\sigma \leq \bar{\sigma}(\beta)$, then both $e_{b}^{*}$ and $e_{r}^{*}$ are interior. If $\sigma>\bar{\sigma}(\beta)$, then we must have $e_{b}^{*}=1$, which implies $e_{r}^{*}=$ $\min \left\{\frac{\sigma(1-2 \beta)}{2}, 1\right\}$. Notice that if $\beta=0.5$, then $\bar{\sigma} \rightarrow \infty$, and the solution is interior for any $\sigma$.

Proof of Proposition 3. The equilibrium promotion gap is

$$
\Delta p(\sigma)=2 \beta \frac{1+2 \beta^{2}\left(1+4 \beta^{2}\right) \sigma^{4}+\left(\frac{3}{2}+2 \beta^{2}\right) \sigma^{2}-2 \sigma}{\left(1+4 \beta^{2} \sigma^{2}\right)^{2}}
$$

Its derivative with respect to $\sigma$ is

$$
\frac{\partial \Delta p}{\partial \sigma}=2 \beta \frac{-2+3\left(1-4 \beta^{2}\right) \sigma+24 \beta^{2} \sigma^{2}+4 \beta^{2}\left(4 \beta^{2}-1\right) \sigma^{3}}{\left(1+4 \beta^{2} \sigma^{2}\right)^{3}}
$$

Define the function $A(\sigma)$ as the numerator of the expression above. $A(\sigma)$ is a third-degree polynomial of $\sigma$, thus, for $\sigma \in \mathbb{R}$, it has three (real or complex) roots $\left(r_{1}, r_{2}, r_{3}\right)$, a local minimum, and a local maximum. Consider its first derivative: $A^{\prime}(\sigma)=3\left(1-4 \beta^{2}\right)+48 \beta^{2} \sigma+12 \beta^{2}\left(4 \beta^{2}-1\right) \sigma^{2}$. The roots for $A^{\prime}(\sigma)=0$ are $\sigma^{m}=\frac{4 \beta-\sqrt{16 \beta^{2}+\left(1-4 \beta^{2}\right)^{2}}}{2 \beta\left(1-4 \beta^{2}\right)}$ and $\sigma^{M}=\frac{4 \beta+\sqrt{16 \beta^{2}+\left(1-4 \beta^{2}\right)^{2}}}{2 \beta\left(1-4 \beta^{2}\right)}$. Notice that $\sigma^{m}<0$ and $\sigma^{M}>0$. At $\sigma=0$, we have $A(0)=-2<0$ and $A^{\prime}(0)=3\left(1-4 \beta^{2}\right)>0$. Thus, $A\left(\sigma^{m}\right)$ must be a local minimum and $A\left(\sigma^{M}\right)$ a local maximum. Thus, $A(\sigma)$ has one negative real $\operatorname{root}\left(r_{1}<\sigma^{m}\right)$, while $r_{2} \leq r_{3}$ must be positive if they are real numbers.

Notice that at $\sigma=1$, the solution is interior, and we have $A(1)=1+8 \beta^{2}+16 \beta^{4}>0$. That is, $\frac{\partial \Delta p}{\partial \sigma}$ is strictly positive at $\sigma=1$. Thus, a real root $r_{2} \in(0,1)$ must exist; $r_{3}>r_{2}$ must also be a real number. We then have that $\frac{\partial \Delta p}{\partial \sigma}<0$ for $\sigma \in\left(0, r_{2}\right), \frac{\partial \Delta p}{\partial \sigma}>0$ for $\sigma \in\left(r_{2}, r_{3}\right)$, and $\frac{\partial \Delta p}{\partial \sigma}<0$ for $\sigma>r_{3}$. Because $\sigma^{M}$ is a local maximum, $\sigma^{M}<r_{3}$. Brute force comparison reveals that $\sigma^{M}>\bar{\sigma}$ for all $\beta \in(0,0.5]$. Thus, $\sigma^{M}$ cannot be an interior solution $\Rightarrow r_{3}>\sigma^{M}$. Thus, in an interior solution, $\frac{\partial \Delta p}{\partial \sigma}<0$ for $\sigma<r_{1}$ and $\frac{\partial \Delta p}{\partial \sigma}>0$ for $\sigma>r_{2}$. We thus have that $\Delta p(\sigma)$ reaches a minimum at $\min \left\{r_{2}, \bar{\sigma}\right\} \equiv \widetilde{\sigma}$.

Proof of Proposition 4. If $\beta=0$, we have that, in an interior solution, $e_{r}=e_{b}=\frac{\sigma}{2}$. The principal's problem is $\max _{\sigma \in[0, \bar{\sigma}(0)]} \theta\left[1-\left(1-\frac{\sigma}{2}\right)^{2}\right]-\sigma$. The FOC for an interior solution is $\theta\left(1-\frac{\sigma}{2}\right)-1=$ 0 . The SOC holds (the problem is globally concave): $-\frac{\theta}{2}<0$. Thus, we have $\sigma^{*}=2 \frac{\theta-1}{\theta}, e^{*}=\frac{\theta-1}{\theta}$. Notice that for all $\theta \geq 1$, the solution is interior, and for all $\theta<1$ the principal does not operate the firm.

Proof of Proposition 5. Notice that the firm can guarantee a non-negative profit by choosing $\sigma=0$. Because the objective function is continuous in $\sigma$ and $[0, \bar{\sigma}(\beta)]$ is a compact set, a maximum always exists. An optimal $\sigma^{*}$ is generically unique because the objective function is a function of polynomials and thus has no flat regions in the interior of $[0, \bar{\sigma}(\beta)]$. The uniqueness here is generic; multiple solutions may arise for measure-zero combinations of parameters $(k, \beta, \theta)$.

Proof of Proposition 6. Define $\sigma(\beta, \theta) \equiv \arg \max _{\sigma \in[0, \sigma(\beta)]} \theta f(\sigma, \beta)-\sigma$, where $f(\sigma, \beta)=e_{b}(\sigma$, $\beta)+e_{r}(\sigma, \beta)-e_{b}(\sigma, \beta) e_{r}(\sigma, \beta)$, where $e_{b}(\sigma, \beta)$ and $e_{r}(\sigma, \beta)$ are given by (5) and (6), respectively. From Proposition 5, the optimal $\sigma$ is generically unique, thus $\sigma(\beta, \theta)$ is well-defined (except perhaps for a measure-zero combination of parameters $(\beta, \theta)$ ). The maximum profit is thus defined as $\Pi(\beta, \theta) \equiv \theta f(\sigma(\beta, \theta), \beta)-\sigma(\beta, \theta)$.

First notice that, for $\sigma(\beta, \theta)>0$, we have that the optimal profit strictly increases with $\theta$ (by the Envelope Theorem):

$$
\begin{equation*}
\frac{\partial \Pi}{\partial \theta}=f(\sigma(\beta, \theta), \beta)>0 \tag{14}
\end{equation*}
$$

To prove Part 1, notice first that at $\theta=0$, trivially, $\sigma(\beta, 0)=0$ and the profit is zero. For $\theta=$ $\bar{\sigma}(\beta)+\varepsilon($ when $\beta<0.5)$, where $\varepsilon>0$, if the principal chooses $\sigma=\bar{\sigma}(\beta)$ we have $f(\bar{\sigma}(\beta), \beta)=$ 1 and the profit is strictly positive. Thus, we know that there exists $\underline{\theta}(\beta)$ such that $\sigma(\beta, \theta)>0$ (and the profit is strictly positive) if and only if $\theta>\underline{\theta}(\beta)$. Now define $\bar{\theta}(\beta)$ as $\bar{\theta}(\beta) \equiv \frac{1}{f_{\sigma}(\bar{\sigma}(\beta), \beta)}$, where $f_{\sigma}$ denotes the derivative with respect to $\sigma$ (note that $f(\sigma, \beta)$ is differentiable in $\sigma$ in the interior of $[0, \bar{\sigma}(\beta)])$. We then have $\sigma(\beta, \bar{\theta}(\beta)+\varepsilon)=\bar{\sigma}(\beta)$ for all $\varepsilon \geq 0$. This proves Part 1 .

To prove Part 2 , consider $\theta \in(\underline{\theta}(\beta), \bar{\theta}(\beta))$, that is, the values for $\theta$ such that $\sigma(\beta, \theta)$ is interior, i.e., $\sigma(\beta, \theta) \in(0, \bar{\sigma}(\beta))$. Thus, the FOC at $\sigma^{*}=\sigma(\beta, \theta)$ must hold: $\frac{\partial \Pi}{\partial \sigma}=\theta f_{\sigma}\left(\sigma^{*}, \beta\right)-$
$1=0$, as well as the second order condition: $\frac{\partial^{2} \Pi}{\partial \sigma^{2}}=\theta f_{\sigma \sigma}\left(\sigma^{*}, \beta\right)<0$. We have that (by implicit differentiation of the FOC): $\frac{\partial \sigma}{\partial \theta}=-\frac{f_{\sigma}\left(\sigma^{*}, \beta\right)}{\theta f_{\sigma \sigma}\left(\sigma^{*}, \beta\right)}=-\frac{1}{\theta^{2} f_{\sigma \sigma}\left(\sigma^{*}, \beta\right)}>0$, proving Part 2. Part 3 follows from (14).

Proof of Proposition 7. For $e_{b}^{*}<1$ (i.e., a strictly interior solution for effort levels), define $f(\sigma, \beta)$ as

$$
f(\sigma, \beta)=e_{b}^{*}+e_{r}^{*}-e_{b}^{*} e_{r}^{*}=\frac{\sigma+4 \beta^{2} \sigma^{2}}{1+4 \beta^{2} \sigma^{2}}-\frac{4 \beta^{2} \sigma^{3}+\left(1-4 \beta^{2} \sigma^{2}\right)\left(\frac{1}{4}-\beta^{2}\right) \sigma^{2}}{\left(1+4 \beta^{2} \sigma^{2}\right)^{2}}
$$

If $e_{b}^{*}<1$ we can write the profit as $\Pi(\sigma, \beta, \theta)=\theta f(\sigma, \beta)-\sigma$. We then have

$$
\frac{\partial \Pi}{\partial \beta}=-\frac{2 \beta \theta \sigma^{2}(\sigma-1)\left\{4 \sigma^{2}(\sigma+1) \beta^{2}-3 \sigma+5\right\}}{\left(4 \sigma^{2} \beta^{2}+1\right)^{3}}
$$

which has non-negative roots at $\beta=0$ and $\beta_{\text {root }}(\sigma)=\frac{1}{2 \sigma} \sqrt{\frac{3 \sigma-5}{\sigma+1}}$. Note that for $\sigma<1, \frac{\partial \Pi}{\partial \beta}$ is strictly positive for $\beta>0$, implying that the optimal bias is $\beta=0.5$. At $\sigma \in\left(1, \frac{5}{3}\right)$, the derivative is strictly negative for $\beta>0$, implying that the optimal bias is $\beta=0$. For $\sigma>5 / 3, \frac{\partial \Pi}{\partial \beta}$ is positive for $\beta<\beta_{\text {root }}(\sigma)$ and negative for $\beta>\beta_{\text {root }}(\sigma)$, implying that the optimal bias is $\beta_{\text {root }}(\sigma)$. Define the following:

$$
\begin{gathered}
\sigma(\beta, \theta) \equiv \arg \max _{\sigma \in[0, \bar{\sigma}(\beta)]} \Pi(\sigma, \beta, \theta), \\
\beta(\sigma, \theta) \equiv \arg \max _{\beta \in[0,0.5]} \Pi(\sigma, \beta, \theta), \text { and } \\
\sigma(\theta) \equiv \arg \max _{\sigma \in[0, \bar{\sigma}(\beta(\sigma, \theta))]} \Pi(\sigma, \beta(\sigma, \theta), \theta) .
\end{gathered}
$$

For now we assume that $\theta$ is such that $\sigma(\theta)<5 / 3$, so that the optimal profit is

$$
\Pi(\theta)=\max \{\Pi(\sigma(0, \theta), 0, \theta), \Pi(\sigma(0.5, \theta), 0.5, \theta)\}
$$

Define $\Delta(\theta) \equiv \Pi(\sigma(0, \theta), 0, \theta)-\Pi(\sigma(0.5, \theta), 0.5, \theta)$, and let $\theta^{\prime}$ denote an element of $\{\theta: \Delta(\theta)=$ $0\}$. We know that at least one such $\theta^{\prime}$ exists because: (i) $\Pi(\sigma(0.5,1), 0.5,1) \geq \Pi(0.5,0.5,1)=$ $0.02>\Pi(\sigma(0,1), 0,1)=0($ see Proposition 4) and (ii) $\Pi(\sigma(0.5,4), 0.5,4)=2<\Pi(\sigma(0,4), 0,4)=$
2.25 .

By continuity there must be a $\theta^{\prime} \in(1,4)$ (numerically, we obtain that $\theta^{\prime} \approx 2.62054$ ) such that $\Pi\left(\sigma\left(0.5, \theta^{\prime}\right), 0.5, \theta^{\prime}\right)=\Pi\left(\sigma\left(0, \theta^{\prime}\right), 0, \theta^{\prime}\right)$. We need to show that $\theta^{\prime}$ is unique. By the Envelope Theorem,

$$
\frac{\partial \Delta(\theta)}{\partial \theta}=f(\sigma(0, \theta), 0)-f(\sigma(0.5, \theta), 0.5)
$$

If $\frac{\partial \Delta\left(\theta^{\prime}\right)}{\partial \theta}>0$ for all $\theta^{\prime} \in\{\theta: \Delta(\theta)=0\}$, then $\theta^{\prime}$ is unique. To show that this is indeed the case, note first that at $\theta^{\prime}$, it must be that $\sigma\left(0.5, \theta^{\prime}\right) \leq 1$, otherwise $\frac{\partial \Pi}{\partial \beta}<0$ and thus $\Pi\left(\sigma\left(0, \theta^{\prime}\right), 0, \theta^{\prime}\right)-$ $\Pi\left(\sigma\left(0.5, \theta^{\prime}\right), 0.5, \theta^{\prime}\right)>0$. Similar reasoning implies that $\sigma\left(0, \theta^{\prime}\right) \geq 1$. We then have that $\Delta\left(\theta^{\prime}\right)=$ 0 implies $f\left(\sigma\left(0, \theta^{\prime}\right), 0\right)-f\left(\sigma\left(0.5, \theta^{\prime}\right), 0.5\right)=\frac{\sigma\left(0, \theta^{\prime}\right)-\sigma\left(0.5, \theta^{\prime}\right)}{\theta^{\prime}}>0$. Thus, $\theta^{\prime}$ is unique. Notice that $\theta^{\prime}<\bar{\theta}$. If not, at $\bar{\theta}$ we have $\Delta(\bar{\theta})<0$, i.e., the optimal bias is $\beta=0.5$. From Proposition 2, $\sigma(0.5, \bar{\theta})>1$. But then $\frac{\partial \Pi}{\partial \beta}$ is strictly negative for $\beta>0$, thus the optimal bias cannot be $\beta=0.5$.

Consider now values for $\theta$ such that $\sigma(\theta) \geq 5 / 3$. In any strictly interior solution for $e_{b}^{*}$, we have $f\left(\sigma, \beta_{\text {root }}(\sigma)\right)=\frac{\sigma^{2}+2 \sigma+25}{32}$, and thus $\Pi\left(\sigma, \beta_{\text {root }}(\sigma), \theta\right)=\theta \frac{\sigma^{2}+2 \sigma+25}{32}-\sigma$ and

$$
\frac{\partial \Pi\left(\sigma, \beta_{\text {root }}(\sigma), \theta\right)}{\partial \sigma}=\theta \frac{2 \sigma+2}{32}-1,
$$

implying that $\Pi\left(\sigma, \beta_{\text {root }}(\sigma), \theta\right)$ has a global minimum at $\sigma=16-\theta$. At any $\sigma \geq \frac{5}{3}$ with $e_{b}^{*}<1$, the principal prefers either to increase or decrease $\sigma$. Thus, there is no strictly interior solution in which $\sigma(\theta) \geq \frac{5}{3}$. That is, $\sigma(\bar{\theta})<\frac{5}{3}$.

# Internet Appendix <br> Subtle Discrimination 

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## OA. 1 Proof of corollaries

Proof of Corollary 1. Note that, from (5) and (6), $e_{r}^{*}=e_{b}^{*} \frac{(0.5+\beta)-2 \beta \sigma(0.5-\beta)}{(0.5-\beta)+2 \beta \sigma(0.5+\beta)}$. It follows that $e_{r}^{*} \geq e_{b}^{*}$ if and only if $\sigma \leq 1$.

Proof of Corollary 2. Solving (7) yields

$$
\begin{align*}
& e_{b}^{*}=\frac{\sigma(0.5-\beta)+(2 \beta-\delta) \sigma^{2}(0.5+\beta-\delta)}{1+(2 \beta-\delta)^{2} \sigma^{2}}  \tag{OA.1}\\
& e_{r}^{*}=\frac{\sigma(0.5+\beta-\delta)-(2 \beta-\delta) \sigma^{2}(0.5-\beta)}{1+(2 \beta-\delta)^{2} \sigma^{2}} \tag{OA.2}
\end{align*}
$$

and $e_{r}^{*}=e_{b}^{*} \frac{(0.5+\beta-\delta)-(2 \beta-\delta) \sigma(0.5-\beta)}{(0.5-\beta)+(2 \beta-\delta) \sigma(0.5+\beta-\delta)}$. It follows that $e_{r}^{*} \geq e_{b}^{*}$ if and only if $\sigma(1-\delta) \leq$ 1.

Proof of Corollary 3. 1. Differentiating (5) with respect to $\sigma$ yields

$$
\frac{\partial e_{b}^{*}}{\partial \sigma}=\frac{0.5-\beta+4 \beta \sigma[(0.5+\beta)-\beta \sigma(0.5-\beta)]}{\left(1+4 \beta^{2} \sigma^{2}\right)^{2}}>0
$$

because $e_{r}^{*} \geq 0$ implies $0.5+\beta-2 \beta \sigma(0.5-\beta) \geq 0 \Rightarrow 0.5+\beta-\beta \sigma(0.5-\beta)>0$.
2. Differentiating (6) with respect to $\sigma$ yields

$$
\frac{\partial e_{r}^{*}}{\partial \sigma}=\frac{(0.5+\beta)-4 \beta \sigma[(0.5-\beta)+\beta \sigma(0.5+\beta)]}{\left(1+4 \beta^{2} \sigma^{2}\right)^{2}}
$$

Note that $\frac{\partial e_{r}^{*}}{\partial \sigma}>0$ for $\sigma=0$ and the numerator is strictly decreasing in $\sigma$ (with limit at $-\infty)$. Solving for the unique positive root for the numerator yields

$$
\sigma_{\text {root }}(\beta) \equiv \frac{\sqrt{(0.5-\beta)^{2}+(0.5+\beta)^{2}}-(0.5-\beta)}{2 \beta(0.5+\beta)}>0
$$

We then define $\widehat{\sigma}(\beta) \equiv \min \left\{\sigma_{\text {root }}(\beta), \bar{\sigma}(\beta)\right\}$.
3.

$$
\frac{\partial \sigma_{\text {root }}(\beta)}{\partial \beta}=\frac{\left(1+2 \beta\left(\frac{1}{2}+2 \beta^{2}\right)^{-\frac{1}{2}}\right)\left(\beta+2 \beta^{2}\right)-(1+4 \beta)\left[\left(\frac{1}{2}+2 \beta^{2}\right)^{\frac{1}{2}}-(0.5-\beta)\right]}{\left(\beta+2 \beta^{2}\right)^{2}} .
$$

The numerator is negative for $\beta=0$ and decreasing in $\beta$ for $\beta \in(0,0.5]$. Thus, we have $\frac{\partial \sigma_{\text {root }}}{\partial \beta}<0$, that is, the region in which $e_{r}^{*}$ declines starts earlier for larger values of $\beta$.

## OA. 2 Remarks on testing and the empirical literature

## OA.2. 1 Testing for discrimination

Our notion of subtle discrimination is relevant in competitive situations, where individuals compete for a prize. The typical test for discrimination is based on comparing the ex-post performances of marginally-treated agents. The gold standard for distinguishing between taste-based discrimination (including stereotypes/incorrect beliefs) and rational statistical discrimination is the "Becker marginal outcome test" (Becker (1957, 1993)). ${ }^{1}$ The idea is that if one group is held to higher standards, its marginally-treated members outperform those from the favored group. Thus, under the null hypothesis of no discrimination (or, more generally, rational statistical discrimination), there should be no group differences in the performance of marginally-treated agents.

Consider a firm or industry where women consistently have lower promotion rates than men. If rational statistical discrimination causes a promotion gap, all else constant, marginally promoted men and women should have similar performances after promotion. In contrast, if marginally-promoted women perform better than marginally-promoted men, biases cause the promotion gap. ${ }^{2}$

In our model, marginally promoted blue and red agents are equally productive. Thus, a well-designed outcome test cannot reject its null hypothesis. Unlike the biases in traditional taste-based or stereotype models, subtle biases are harder to detect with an outcome test. If workers are close substitutes, a slight bias towards one group may not harm a firm's profit. Thus, our model implies that subtle discrimination should feature alongside statisti-

[^13]cal discrimination as the null hypothesis in Becker outcome tests in promotion contexts.

## OA.2.2 Breaking ties and testing for subtle discrimination

Our main model assumes that "ties" (i.e., individuals with very similar qualifications and performance records) are unexceptional. In practice, ties between candidates in terms of qualifications are ubiquitous because evaluation scales are often discrete, e.g., the 9-box performance-by-potential grid (Effron and Ort (2010)). Furthermore, Frederiksen et al. (2017) find that performance scales tend to be restricted, with five- or six-point scales being the norm. Ties are also likely when candidates' qualifications are assessed across several domains and candidates excel in different areas, i.e., there is no clear winner across all relevant qualifications. Similarly, ties are likely when candidates' scores are aggregated across several decision-makers (such as the members of a hiring committee), even if individual members avoid ties when ranking candidates. Averaging also makes group decisions less variable than individual ones (Adams and Ferreira (2010)) and thus reduces the perceived differences between candidates. When ties occur, they are often broken based on subjective criteria, allowing subtle biases to affect decisions.

A leading example of the relevance of "tie-breaking" is academic co-authorship. Sarsons et al. (2021) show that while both men and women benefit equally from solo authorship, co-authorship harms women's chances of being tenured. Consistent with our notion of subtle discrimination, employers are likelier to "break the tie" in favor of male co-authors when trying to attribute credit for joint work. See Heilman and Haynes (2005) for further evidence of gender bias in team credit attribution.

An approach to testing for subtle discrimination is to consider its impact on discriminated individuals (e.g., as in Hengel (2022)). In our context, this involves comparing agents' investment choices under different stakes. An instructive example - although in a somewhat different context - is the work of Filippin and Guala (2013), who conducted all-pay auctions in the lab and find that auctioneers, despite incentives to pick the highest bidder, often favored their own group in tie situations. In response, out-group bidders reduced their bids, widening the earnings gap between groups.

Lab experiments are especially effective in directly testing for subtle discrimination. Foschi et al. (1994) designed an experiment where subjects must promote at most one of two candidates. When subjects choose between a pair of candidates of the same sex, sometimes no one is promoted. Thus, the authors can infer the minimum threshold of
qualifications for promotions for each sex. They find that subjects use similar thresholds for male and female candidates. That is, men and women are held to the same standards when competing against someone of the same sex. By contrast, when men and women with similar qualifications compete against each other, subjects are more likely to promote men. In a similar experimental study where subjects have to hire one out of two candidates to perform a task, Barron et al. (2022) find that subjects discriminate against women when a male and a female candidate are either identically or differently qualified. However, they do not discriminate when one candidate is clearly more qualified than the other. This evidence is consistent with our definition of subtle discrimination but inconsistent with statistical discrimination. In related work, Reuben et al. (2014) and Moss-Racusin et al. (2012) show experimental evidence that subjects’ preexisting subtle biases explain their propensity to hire male candidates when choosing between candidates with similar qualifications.

## OA.2.3 Related empirical literature

Hospido et al. (2019), Bosquet et al. (2019) and Azmat et al. (2020) show that, in highstakes environments women have lower promotion rates, partially because they often do not seek promotions in the first place. Our model links this discouragement to subtle discrimination, which has greater impact in high-stakes careers. Moreover, several recent papers provide suggestive evidence of subtle discrimination. Benson et al. (2021) find that despite similar performance both before and after promotions, women consistently get lower "potential" ratings than men. Women are also more likely to be fired for professional misconduct (Egan et al. (2022)), and receive less credit for workplace innovation (Luksyte et al. (2018)) and experience (Cziraki and Robertson (2021)).

Our results also speak to the literature on the gender gap in willingness to compete (Niederle and Vesterlund (2007); see also Niederle and Vesterlund (2011) for a review). Our model predicts that women are less willing to compete against men than against other women. Using a lab experiment, Geraldes (2020) shows that when given an opportunity to choose a competitor's gender, women are as likely to enter a competition as men are. In line with our predictions about the effect of stakes, Buser et al. (2023) show that women are less willing to compete against men in a high-stakes TV game show.

## OA. 3 Continuous skill levels

Our model is not limited to scenarios where skill levels are discrete and workers can be equally qualified with positive probability. Subtle discrimination may occur whenever a decision-maker can credibly claim to use private information for choosing between candidates. This holds true even when observable differences in skills are large, as long as the decision-maker can plausibly deny being biased.

Suppose agents can choose to acquire a skill level in the unit interval, $s_{i} \in[0,1]$. For simplicity, we assume that $e_{i}=s_{i}$, i.e., effort deterministically translates into skill. Thus, the difference in observable skill is $\Delta s=e_{r}-e_{b}$. As in Section 2, let $F$ (.) denote the c.d.f. of $\Delta x$. For simplicity, assume that $\underline{x}=-1, \bar{x}=1, F($.$) is uniform, and \omega=1$. That is, for any differences in skill $\Delta s$, the principal can plausibly justify promoting Blue. Thus, subtle discrimination may occur for any $\Delta s .^{3}$

Consider first how an unbiased decision-maker would choose between the two candidates. For given $e_{b}$ and $e_{r}$, the probability that Blue is chosen is given by $P\left(e_{b}, e_{r}\right)=$ $\frac{1}{2}\left(1+e_{b}-e_{r}\right)$. In the contest literature, $P\left(e_{b}, e_{r}\right)$ is known as the contest success function, or CSF. Below we show that the key to our results is how subtle biases affect the CSF.

Under the unbiased CSF, the agents' expected utilities are (assuming $k=2$ for simplicity) $U_{b}=\sigma \frac{1}{2}\left(1+e_{b}-e_{r}\right)-e_{b}^{2}$ and $U_{r}=\sigma\left[1-\frac{1}{2}\left(1+e_{b}-e_{r}\right)\right]-e_{r}^{2}$, leading to first-order conditions $\frac{\sigma}{2}-e_{b}^{*}=0$ and $\frac{\sigma}{2}-e_{r}^{*}=0$, i.e., $e_{b}^{*}=e_{r}^{*}=\frac{\sigma}{2}$, as in our main model.

Suppose now the principal is subtly biased. Then, we write the CSF as $P\left(e_{b}, e_{r}\right)=$ $\min \left\{\frac{1}{2}\left(1+e_{b}-e_{r}\right)+b\left(e_{b}, e_{r}\right), 1\right\}$, where $b\left(e_{b}, e_{r}\right)$ is the bias in favor of Blue. The bias $b\left(e_{b}, e_{r}\right)$ may depend on the observed skill levels because the cost to the principal from making a biased decision may depend on the skill levels. How subtle discrimination distorts effort levels thus depends on the particular functional form of $b\left(e_{b}, e_{r}\right)$.

Consider first the case in which $b\left(e_{b}, e_{r}\right)=\beta e_{b}$, for $\beta \in(0,0.5]$. This may be the case when the principal finds it easier to justify choosing Blue when Blue is more objectively qualified. Assuming an interior solution, the first-order conditions are $\sigma\left(\frac{1}{2}+\beta\right)-e_{b}^{*}=0$ and $\frac{\sigma}{2}-e_{r}^{*}=0$, which implies $e_{b}^{*}>e_{r}^{*}$. (An interior solution always obtains for $\sigma<\frac{2}{1+2 \beta}$ ). Thus, subtle discrimination produces an encouragement effect for Blue. If $\sigma>\frac{2}{1+2 \beta}$, the equilibrium is $e_{b}^{*}=1$ and $e_{r}^{*}=0$. That is, in high-stakes situations, subtle discrimination leads to a discouragement effect for Red.

[^14]Suppose now that $b\left(e_{b}, e_{r}\right)=\beta\left(1-e_{r}\right)$, for $\beta \in(0,0.5]$. Intuitively, this is the case when the principal finds it easier to justify choosing Blue when Red is less objectively qualified. Assuming an interior solution, the first-order conditions are $\frac{\sigma}{2}-e_{b}^{*}=0$ and $\sigma\left(\frac{1}{2}+\beta\right)-e_{r}^{*}=0$, which implies $e_{b}^{*}<e_{r}^{*}$. Thus, subtle discrimination produces an overcompensation effect for Red. (Again, an interior solution always obtains for $\sigma<\frac{2}{1+2 \beta}$ ).

These examples show that with continuous skill levels and subtle discrimination, either discouragement or overcompensation effects can arise depending on the chosen bias function $b\left(e_{b}, e_{r}\right)$. While the above simple functions produce only one effect at a time, complex ones can yield either, depending on parameters. However, justifying more complex functions based only on intuition or introspection is challenging. Our main model with discrete skills and arbitrarily small subtle bias $(\omega \rightarrow 0)$ provides a microfoundation for the bias function

$$
\begin{equation*}
b\left(e_{b}, e_{r}\right)=\beta\left(1-e_{b}-e_{r}+2 e_{b} e_{r}\right) \tag{OA.3}
\end{equation*}
$$

Thus, we can replicate all the results in the paper using the continuous approach developed here if we use the bias function in Eq. (OA.3).

To conclude, we show that the equilibrium effort levels depend on how the subtle bias affects the contest success function. Our main model shows a natural example where the subtle bias may lead to either discouragement or overcompensation. Importantly, our model has predictions for when each of these effects is likely to dominate. The simple examples in this Appendix section show that models with continuous skills and non-infinitesimal subtle biases can also generate overcompensation and discouragement.

## OA. 4 Skills not fully firm-specific

Here we show that the skill does not have to be fully firm-specific for our results to go through. Suppose that by acquiring the skill an agent improves her outside wage by $w_{0}>0$, such that $W>w_{0}$, i.e., the skill is still more valuable to the worker inside the firm than outside. Then, the equilibrium investment levels (assuming an interior solution) are

$$
\begin{align*}
& e_{b}^{*}=\frac{\sigma(0.5-\beta)+2 \beta \sigma^{2}(0.5+\beta)+\sigma \sigma_{0}\left(0.25-\beta^{2}\right)}{1+4 \beta^{2} \sigma\left(\sigma-\sigma_{0}\right)-\sigma_{0}^{2}\left(0.25-\beta^{2}\right)} ;  \tag{OA.4}\\
& e_{r}^{*}=\frac{\sigma(0.5+\beta)-2 \beta \sigma^{2}(0.5-\beta)+\sigma \sigma_{0}\left(0.25-\beta^{2}\right)}{1+4 \beta^{2} \sigma\left(\sigma-\sigma_{0}\right)-\sigma_{0}^{2}\left(0.25-\beta^{2}\right)}, \tag{OA.5}
\end{align*}
$$

where $\sigma_{0}=w_{0} / k$. Note that Corollary 1 is unchanged: $e_{r}^{*} \geq e_{b}^{*}$ if and only if $\sigma \leq 1$.
Even if the skill is fully general, it could be more valuable to the worker inside the firm if there are switching (or search) costs. Alternatively, in the presence of other realistic frictions, skill specificity does not matter, as we show in the next section of this appendix.

## OA. 5 Market equilibrium under perfect competition

In this section, we show the robustness of the main results to different assumptions. The modification presented here has the following important features: (i) Firms compete for workers and, thus, have no bargaining power; (ii) Workers invest in general skills before being hired and their skills are observable to all; (iii) Compensation may depend on skills. We consider a stylized labor market with search frictions and asymmetric employer learning. Despite such frictions, in the absence of a subtle bias $\beta$, the first-best investment levels are achieved. We then show that a positive subtle bias distorts investment decisions in a similar way as in the main model. Unlike the main model, firms have zero profits in equilibrium regardless of their biases. Thus, they have no incentives to change their biases, i.e., market forces do not eliminate biases.

Consider an economy with mass $L$ of agents, half of them blue and the other half red. Each agent lives for two periods (we can think of overlapping generations that enter the market in each period with mass $L$ ). There is a mass $F$ of firms with infinite lifespans. Each firm is endowed with a technology that allows them to employ two workers in each period. Workers can be skilled, $s_{i}=1$, or unskilled, $s_{i}=0$. Skill is observed at the time of hiring and is general in the sense that it is valuable to all companies.

After an agent works for a firm for one period, the firm learns about her managerial ability, $m_{i} \in\{0,1\}$. As in asymmetric employer learning models (see, e.g., Waldman (1984) and Greenwald (1986)), this information is private to the firm. After learning the managerial abilities of its two workers, the firm decides which worker (if any) to promote to the top position, job 2 . We assume that promoting an agent with no managerial ability leads to large losses. Thus, firms never promote a worker unless they are sure that $m_{i}=1$. Non-promoted agents remain at job 1 or leave the firm (for simplicity, we assume that the productivity of an "old" worker is zero at job 1). Formally, with probability $\mu$, agent $i$ generates a signal $m_{i}=1$, indicating suitability for managerial positions. Among those with $m_{i}=1$, agents can be skilled $\left(s_{i}=1\right)$ or unskilled $\left(s_{i}=0\right)$. Promoting an unskilled agent increases the principal's expected payoff by $l>0$ while promoting a skilled agent increases
the payoff by $l+H$, where $H>0$. That is, a skilled agent is always more productive than an unskilled one once assigned to job 2. Note that the main model in the paper is a special case of this one in which $\mu=1$.

The timing is as follows. Within Period 1 (when agents are young), there are three dates. At Date 0 , young agents first decide how much to invest in skills, $e_{i} \in[0,1]$ at cost $c\left(e_{i}\right)=\frac{e_{i}^{2}}{2}$ (i.e., we set $k=1$ for simplicity). As in the main model, $e_{i}$ is the probability of acquiring the skill.

At Date 1, each firm searches for workers to fill its two job 1 vacancies. Search is costly in the sense that each firm can find only two workers per period. Each worker can be matched with multiple firms at this stage; workers join firms that offer them the best conditions. Workers cannot observe the identities of agents who hold job offers. We assume that $2 F>L$, that is, there are more vacancies than workers. Random search implies that each worker receives more than one offer to choose from. Firms offer an entry-level wage $w_{1}\left(s_{i}\right)$ to a candidate with skill $s_{i}$, conditional on both candidates accepting the offer. At this stage, there is no need for firms to promise a wage upon promotion in Period 2; this wage is determined later in the labor market. Firms must hire two workers, otherwise, they cannot produce.

At Date 2, after $m_{i}$ is internally revealed to each firm, firms make their promotion decisions. Promotion decisions are visible, thus firms with vacancies can make poaching offers $w_{p}\left(s_{i}\right)$ to promoted workers from other firms. To retain such workers, incumbent firms must match any poaching offer. ${ }^{4}$ Non-promoted workers are never poached because they might have no managerial abilities.

Finally, in Period 2, promoted agents work at job 2 and receive wages $w_{p}\left(s_{i}\right)$ and nonpromoted workers stay at job 1 and continue receiving wage $w_{1}\left(s_{i}\right)$. For simplicity, we assume no discounting between periods.

We characterize the equilibrium by working backwards. At the end of Date 2, mass $\frac{(1-\mu)^{2} L}{2}$ of active firms have vacancies because none of their workers has managerial ability. Such firms try to poach promoted workers from other firms. Thus, workers with managerial abilities are in short supply; their wages must be $w_{p}\left(s_{i}=1\right)=l+H$ and $w_{p}\left(s_{i}=0\right)=l$. This implies that all active firms make zero profit when employing old workers in job 2. We assume that firms do not overtly discriminate and, thus, always promote skilled agents ahead of unskilled agents.

At Date 1, firms anticipate they will make zero profits from either type of worker and

[^15]are thus indifferent between $s_{i}=0$ and $s_{i}=1$ workers. This implies that they are also indifferent between blue and red workers. Firms then offer $w_{1}\left(s_{i}\right)=0$ to all workers as the base wage.

At Date 0 , workers make investment decisions knowing that (i) wages upon promotion are $w_{p}\left(s_{i}=1\right)=l+H$ and $w_{p}\left(s_{i}=0\right)=l$, (ii) the probability of being paired with either blue or red agents is 0.5 , and (iii) firms have a subtle bias $\beta$ towards blue agents (or, equivalently, $2 \beta$ is the proportion of firms with subtle bias 0.5 ).

In a rational expectations equilibrium, agents maximize utility taking the equilibrium effort of all other blue and red agents as given, $e_{b}^{*}$ and $e_{r}^{*}$, here conjectured to be the same for all agents of the same type. A blue agent's expected utility is:

$$
\begin{align*}
& \mu(1-\mu)\left(l+e_{b} H\right)+ \\
& \frac{\mu^{2}}{2}\left[(l+H) e_{b}\left(1-e_{r}^{*}\right)+(l+H)\left(\frac{1}{2}+\beta\right) e_{b} e_{r}^{*}+l\left(\frac{1}{2}+\beta\right)\left(1-e_{b}\right)\left(1-e_{r}^{*}\right)\right]+ \\
& \frac{\mu^{2}}{2}\left[(l+H) e_{b}\left(1-e_{b}^{*}\right)+(l+H) \frac{1}{2} e_{b} e_{b}^{*}+l \frac{1}{2}\left(1-e_{b}\right)\left(1-e_{b}^{*}\right)\right]-\frac{e_{b}^{2}}{2}, \tag{OA.6}
\end{align*}
$$

and a red agent's expected utility is:

$$
\begin{align*}
& \mu(1-\mu)\left(l+e_{r} H\right)+ \\
& \frac{\mu^{2}}{2}\left[(l+H) e_{r}\left(1-e_{b}^{*}\right)+(l+H)\left(\frac{1}{2}-\beta\right) e_{r} e_{b}^{*}+l\left(\frac{1}{2}-\beta\right)\left(1-e_{r}\right)\left(1-e_{b}^{*}\right)\right]+ \\
& \frac{\mu^{2}}{2}\left[(l+H) e_{r}\left(1-e_{r}^{*}\right)+(l+H) \frac{1}{2} e_{r} e_{r}^{*}+l \frac{1}{2}\left(1-e_{r}\right)\left(1-e_{r}^{*}\right)\right]-\frac{e_{r}^{2}}{2} . \tag{OA.7}
\end{align*}
$$

Note that if the premium is the same for all skill levels $(H=0), \mu=1$, and workers pair with a different type with probability one, we are back in the case of the main model.

The first-order conditions are:

$$
\begin{aligned}
& \mu(1-\mu) H+\frac{\mu^{2}}{2}\left[2(l+H)-l(1+\beta)+e_{r}^{*}\left(l\left(\frac{1}{2}+\beta\right)-(l+H)\left(\frac{1}{2}-\beta\right)\right)-H \frac{e_{b}^{*}}{2}\right]-e_{b}=0 \\
& \mu(1-\mu) H+\frac{\mu^{2}}{2}\left[2(l+H)-l(1-\beta)+e_{b}^{*}\left(l\left(\frac{1}{2}-\beta\right)-(l+H)\left(\frac{1}{2}+\beta\right)\right)-H \frac{e_{r}^{*}}{2}\right]-e_{r}=0
\end{aligned}
$$

Note that while the reaction function of red agents is always downward-sloping, the reaction function of blue agents is upward-sloping if and only if $2 l \beta>H(0.5-\beta)$. This is always true for sufficiently high $\beta$. Thus, a large subtle bias is sufficient for the blue agents'
reaction functions to slope upwards.
We can rewrite the equilibrium conditions as

$$
\begin{align*}
\sigma_{l}\left(1-\beta+2 \beta e_{r}^{*}\right)+\sigma_{h}\left[2-(0.5-\beta) \mu e_{r}^{*}\right] & =e_{b}^{*}  \tag{OA.8}\\
\sigma_{l}\left(1+\beta-2 \beta e_{b}^{*}\right)+\sigma_{h}\left[2-(0.5+\beta) \mu e_{b}^{*}\right] & =e_{r}^{*} \tag{OA.9}
\end{align*}
$$

where $\sigma_{l}=\frac{2 \mu^{2} l}{4+\mu^{2} H}$ and $\sigma_{h}=\frac{2 \mu H}{4+\mu^{2} H}$. The unique solution (if interior) is given by

$$
\begin{align*}
& e_{b}^{*}=\frac{\sigma_{l}(1-\beta)+2 \sigma_{h}+\left[2 \beta \sigma_{l}-\mu(0.5-\beta) \sigma_{h}\right]\left[\sigma_{l}(1+\beta)+2 \sigma_{h}\right]}{1+\left[2 \beta \sigma_{l}-\mu(0.5-\beta) \sigma_{h}\right]\left[2 \beta \sigma_{l}+\mu(0.5+\beta) \sigma_{h}\right]} ;  \tag{OA.10}\\
& e_{r}^{*}=\frac{\sigma_{l}(1+\beta)+2 \sigma_{h}-\left[2 \beta \sigma_{l}+\mu(0.5+\beta) \sigma_{h}\right]\left[\sigma_{l}(1-\beta)+2 \sigma_{h}\right]}{1+\left[2 \beta \sigma_{l}-\mu(0.5-\beta) \sigma_{h}\right]\left[2 \beta \sigma_{l}+\mu(0.5+\beta) \sigma_{h}\right]} . \tag{OA.11}
\end{align*}
$$

We have that $e_{r}^{*} \geq e_{b}^{*}$ if and only if $2 \sigma_{l}\left(1-2 \sigma_{l}\right) \geq\left[(8+\mu) \sigma_{l}+4 \mu \sigma_{h}\right] \sigma_{h}$. To interpret this condition, note that, in this version of the model, we have two measures for the stakes: $l$ and $H$, which are proportional to $\sigma_{l}$ and $\sigma_{h}$. If $\sigma_{l}>0.5$, the condition for $e_{r}^{*} \geq e_{b}^{*}$ does not hold. If $\sigma_{l}<0.5$, the condition holds only if $\sigma_{h}$ is sufficiently small. Thus, as in the main model, the overcompensation effect dominates for low stakes (low $\sigma_{l}$ and $\sigma_{h}$ ), while the discouragement effect dominates for high stakes (high $\sigma_{l}$ and/or high $\sigma_{h}$ ).

How do we know that there exist parameters that lead to either case? Notice that if $H \rightarrow 0$, the unique (interior) equilibrium converges to

$$
\begin{equation*}
e_{b}^{*}=\frac{\sigma_{l}(1-\beta)+2 \beta \sigma_{l}^{2}(1+\beta)}{1+4 \beta^{2} \sigma_{l}^{2}} \text { and } e_{r}^{*}=\frac{\sigma_{l}(1+\beta)-2 \beta \sigma_{l}^{2}(1-\beta)}{1+4 \beta^{2} \sigma_{l}^{2}} \tag{OA.12}
\end{equation*}
$$

with the condition for Red to invest more becoming simply $\sigma_{l} \leq 0.5$, which is equivalent to $\mu^{2} l \leq 1$. If $\mu=1$ as in the main model, the condition is thus identical to Corollary 1 , because $\sigma$ in this context is identical to $l=W / k$.

## OA. 6 Filling all slots and promoting low-skill workers

In the previous section, we showed an example in which job-2 slots are sometimes left vacant. This has no effect on our analysis, as long as low-skilled workers have some probability of being promoted.

In the main model, our assumption that $l>0$ implies that the principal always prefers to promote a worker instead of leaving the post vacant (or employing a worker from the
outside). Suppose instead that $l<0$. Then, both reaction functions are downward sloping, and the equilibrium investment levels (assuming an interior solution) are

$$
\begin{equation*}
e_{b}^{*}=\frac{\sigma-\sigma^{2}(0.5-\beta)}{1-\sigma^{2}\left(0.25-\beta^{2}\right)} \quad \text { and } \quad e_{r}^{*}=\frac{\sigma-\sigma^{2}(0.5+\beta)}{1-\sigma^{2}\left(0.25-\beta^{2}\right)} \tag{OA.13}
\end{equation*}
$$

Note that $e_{b}^{*}>e_{r}^{*}$ in any interior solution. Thus, for the overcompensation effect to dominate, we need that when both agents are unskilled, the firm promotes from the inside with positive probability. Intuitively, Red's overcompensation only pays off when Blue choose low effort. If Blue cannot be promoted unless he has the skill, Blue does not slack off, and the overcompensation effect is always dominated by the discouragement effect.

## OA. 7 General functional form of investment cost

Here, we show that our main results hold when we use a more general cost function $c(e)$, such that $c(0)=0, c(1) \rightarrow \infty, c^{\prime}()>0,. c^{\prime \prime}()>$.0 . In particular, we (numerically) solve the agents' and the principal's problems for the following cost function $c(e)=\frac{k}{\alpha} \frac{e^{\alpha}}{1-e^{\gamma}}$.

This form has several advantages. First, for $\alpha=2$ and $\gamma \rightarrow \infty$, it converges to the quadratic cost function $\frac{k e^{2}}{2}$ used in the main text. Second, for the agent's problem, it guarantees an interior solution for any value of the premium-cost ratio, $\sigma \equiv \frac{W}{k}$. Finally, we confirm that for a sizable interval of parameters values $\alpha$ and $\gamma$ and for any value of the productivity-cost ratio $\theta$, the social welfare is maximized when both agents are treated symmetrically, that is, invest in their human capital. In particular, we define social surplus under the asymmetric treatment (only one agent invests in her human capital) as $s_{1 a}(\theta ; \alpha, \gamma)=\max _{e} \theta e-\frac{k}{\alpha} \frac{e^{\alpha}}{1-e^{\gamma}}$ and under the symmetric treatment (both agents invest) as $s_{2 a}(\theta ; \alpha, \gamma)=\max _{e} \theta e(2-e)-2 \times \frac{k}{\alpha} \frac{e^{\alpha}}{1-e^{\gamma}}$.

Then, for $\alpha \in[2.0,10.0]$, we compute $\bar{\gamma}$ such that for all $\gamma<\bar{\gamma}$ (see Figure OA.1), the social welfare is maximized when both agents invest in their human capital for all values of $\theta$. That is, for all values of $\theta, s_{2 a}(\theta ; \alpha, \gamma)>s_{1 a}(\theta ; \alpha, \gamma)$ as long as $\alpha \in[2.0,10.0]$ and $\gamma<\bar{\gamma}$. Thus, we confirm that our results are not driven by the fact that under a quadratic cost function, for $\theta>1$ it is more socially efficient to treat agents asymmetrically. In the remainder of this section we assume $\alpha=2.0(\bar{\gamma} \approx 18.5013$ for $\alpha=2.0)$.

Figure OA. 2 shows the equilibrium investment levels as a function of the premium-cost ratio, $\sigma$, for two levels of the subtle bias, $\beta \in\{0.1,0.4\}$ and the following levels of the cost function parameters, $\gamma \in\{0.5,9.0,18.0\}$. Note that for $\alpha=2.0$, all these values of $\gamma$ are


Figure OA.1: $\bar{\gamma}$ as a function of the cost function parameter $\alpha$
below $\bar{\gamma}$. Figure OA. 2 confirms the results in Proposition 2 and Figure 1 of the main text. In particular, it shows that for low values of $\sigma$, Red invests more than Blue, while for high values of $\sigma$, it is Blue who invests more. Therefore, the existence of the overcompensation and the discouragement effects is not driven by our choice of the quadratic cost function.


Figure OA.2: Equilibrium investments of blue and red agents, $e_{b}^{*}$ and $e_{r}^{*}$, as functions of the premium-cost ratio, $\sigma$, for different values of the subtle bias $\beta$ and the cost function parameter $\gamma$.

Figure OA. 3 shows the equilibrium promotion gaps for the same values of $\beta$ and $\gamma$ as functions of the premium-cost ratio $\sigma$ (it replicates the results in Proposition 3 and Figure 2 of the main text). Again, we confirm that for a wide range of parameters, the promotion
gap has U-shape and that under high values of $\sigma$, the contribution of the achievement gap to the promotion gap is lower than under low values of $\sigma$.

Figure OA. 4 shows the solution for the principal's problem, the optimal premium-cost ratio, $\sigma^{*}(\theta ; \beta, \gamma, \alpha)$, the optimal profit, $\Pi^{*}(\theta ; \beta, \gamma, \alpha)$, and the resulting promotion gap, $\Delta p^{*}(\theta ; \beta, \gamma, \alpha)$ as functions of the productivity-cost ratio $\theta$ and for several values of the subtle bias $\beta$ and parameters $\gamma$ and $\alpha, \beta \in\{0.1,0.4\}, \gamma \in\{0.5,9.0,18.0\}$ and $\alpha=2.0$.


Figure OA.3: Equilibrium promotion gap, $\Delta p^{*}$, as a function of the premium-cost ratio, $\sigma$, for different values of the subtle bias $\beta$ and the cost function parameter $\gamma$.

Next, we consider a case when a firm optimally chooses its subtle bias. We confirm that there exists $\theta^{\prime}$ such that for all $\theta<\theta^{\prime}$, the firm prefers $\beta^{*}=0.5$ and for all $\theta>\theta^{\prime}$, the firm prefers $\beta^{*}=0$. Figure OA. 5 shows $\theta^{\prime}$ as a function of $\gamma$, where $\gamma \in[0.5,18.0]$ and $\alpha=2$. Thus, firm polarization occurs even under a more general cost function.

Finally, in Figure OA. 6 we replicate the results in Figure 4 from the main text for $\gamma=9.0$ and $\alpha=2.0$. For other values of the cost function parameters, $\gamma$ and $\alpha$, the optimal stake, $\sigma^{*}$, the resulting profit, $\Pi^{*}$, and the promotion gap, $\Delta p^{*}$ have similar shapes.


Figure OA.4: Optimal premium-cost ratio, $\sigma^{*}$, firm profit, $\Pi^{*}$, and promotion gap, $\Delta p^{*}$, as a function of the productivity-cost ratio, $\theta$, for different values of the subtle bias $\beta$ and the cost function parameter $\gamma$.

## OA. 8 Non-binary skill

Here we show that our results are robust to relaxing the assumption that the acquired skill is a binary variable. We extend the model to three levels of skill, $s_{i}=\{0,0.5,1\}$.

## OA.8.1 Agent's problem

As in the main model, at Date 1, the agents simultaneously undertake a nonverifiable investment (or effort), $e_{i} \in[0,1], i \in b, r$, in firm-specific human capital. Both agents are risk-neutral and have the same skill-acquisition cost function, $c\left(e_{i}\right)$, strictly increasing and convex and such that $c(0)=0$. Agent $i$ 's probability of acquiring the lower skill level, $s_{i}=0.5$, is $e_{i}$ and the higher skill level, $s_{i}=1$, is $\alpha e_{i}$, where $\alpha \in[0,1)$. For example, in project management, project planning, scheduling, and budgeting are considered parts of the foundational skill level ( $s_{i}=0.5$ in our interpretation), while vision and goal-setting for projects as well as ability to align project objectives with organizational strategy are considered parts of more advanced skill level ( $s_{i}=1$ in our model). For completeness,


Figure OA.5: $\theta^{\prime}$ as a function of the cost function parameter $\gamma$


Figure OA.6: Optimal premium-cost ratio, equilibrium profit, and promotion gap, as functions of the productivity-cost ratio, $\theta$, for $\gamma=9.0$ and $\alpha=2.0$.
we assume that the probability of an agent remaining unskilled is $1-e_{i}-\alpha e_{i} \geq 0$, which implies that the following condition is necessary for an interior solution: $e_{i} \leq \frac{1}{1+\alpha}$.

First, we compute probabilities of three disjoint outcomes:

1. Both agents are equally skilled with probability $p_{t i e}=e_{b} e_{r}+\alpha^{2} e_{b} e_{r}+\left(1-\alpha e_{b}-\right.$ $\left.e_{b}\right)\left(1-\alpha e_{r}-e_{r}\right)=1+2 e_{b} e_{r}\left(1+\alpha+\alpha^{2}\right)-\left(e_{b}+e_{r}\right)(1+\alpha)$;
2. Blue is more skilled with probability $p_{b t o p}=\alpha e_{b}\left(1-\alpha e_{r}\right)+e_{b}\left(1-e_{r}-\alpha e_{r}\right)$;
3. Red is more skilled with probability $p_{r t o p}=\alpha e_{r}\left(1-\alpha e_{b}\right)+e_{r}\left(1-e_{b}-\alpha e_{b}\right)$.

Now we can write down the maximization problem for Blue and Red under the assumption that the effort cost function is quadratic, $c\left(e_{i}\right)=\frac{k e_{i}^{2}}{2}$ :

$$
\begin{align*}
& \max _{e_{b}} \sigma\left(p_{\text {btop }}+\left(\frac{1}{2}+\beta\right) p_{\text {tie }}\right)-\frac{e_{b}^{2}}{2}  \tag{OA.14}\\
& \max _{e_{r}} \sigma\left(p_{\text {rtop }}+\left(\frac{1}{2}-\beta\right) p_{\text {tie }}\right)-\frac{e_{r}^{2}}{2} . \tag{OA.15}
\end{align*}
$$

The first order conditions imply the following reaction functions for Blue and Red $e_{b}=$ $\sigma\left((\alpha+1)\left(\frac{1}{2}-\beta\right)+2 \beta e_{r}\left(\alpha^{2}+\alpha+1\right)\right)$ and $e_{r}=\sigma\left((\alpha+1)\left(\frac{1}{2}+\beta\right)-2 \beta e_{r}\left(\alpha^{2}+\alpha+1\right)\right)$. Note that in the absence of subtle bias $\beta=0$, the reaction functions are flat, just as in the main text: $e_{b}^{*}=e_{r}^{*}=\frac{\sigma}{2}(\alpha+1)$.

If $\alpha=0$, the optimal investment levels are the same as in the main text (see Eq. (8) and (9) in the main text). For $\alpha \in(0,1)$, we solve for $e_{b}^{*}$ and $e_{r}^{*}$ and obtain the solution:

$$
\begin{align*}
& e_{b}^{*}=\sigma(1+\alpha) \frac{0.5-\beta+2 \beta \sigma(0.5+\beta)\left(1+\alpha+\alpha^{2}\right)}{1+4 \sigma^{2} \beta^{2}\left(\alpha^{2}+\alpha+1\right)^{2}}  \tag{OA.16}\\
& e_{r}^{*}=\sigma(1+\alpha) \frac{0.5+\beta-2 \beta \sigma(0.5-\beta)\left(1+\alpha+\alpha^{2}\right)}{1+4 \sigma^{2} \beta^{2}\left(\alpha^{2}+\alpha+1\right)^{2}} \tag{OA.17}
\end{align*}
$$

The equilibrium investment levels of Blue and Red for $\alpha \in(0,1)$ have the same functional form as in the main version of the model. Figure OA. 7 shows the equilibrium investment levels of Blue and Red as functions of the premium-cost ratio, $\sigma$, for two levels of the subtle bias $\beta$ and parameter $\alpha$. As in the main text, in low-stakes situations, the overcompensation effect dominates and in high-stakes situations, the discouragement effect dominates. Therefore, firms that offer low-stakes careers (or less human-capitalintensive) find it profitable to engage in subtle discrimination in order to benefit from the overcompensation effect. In contrast, firms that offer high-stakes careers (and are more human-capital-intensive) prefer not to discriminate.

## OA.8.2 Principal's problem

Here, we show that even under a finer skill assessment, the principal prefers high levels of subtle discrimination in cases of low productivity and low levels of subtle discrimination in cases of high productivity. We assume that upon promoting a moderately skilled agent with $s_{i}=0.5$, the principal's profit increases by $H$, as stated in the main text. Similarly, when promoting a highly skilled agent with $s_{i}=1$, the profit increases by $\mu H$, where $\mu \geq 1$. Then, the principal's profit is:

$$
\begin{equation*}
\max _{w_{l} \geq 0, w_{l}+W \geq 0} l+\mu H \cdot P\left(s_{b \vee r}=1\right)+H \cdot P\left(s_{b \vee r}=0.5\right)-W-2 w_{l} \tag{OA.18}
\end{equation*}
$$



Figure OA.7: Equilibrium investments of blue and red agents, $e_{b}^{*}$ and $e_{r}^{*}$, as functions of the premium-cost ratio, $\sigma$, for different values of the subtle bias $\beta$ and the parameter $\alpha$.

When $w_{l}=0$ and the solution is interior, that is $\sigma \leq \bar{\sigma}(\alpha, \beta)$, we can rewrite the principal's profit maximization problem as:

$$
\begin{equation*}
\pi(\alpha, \mu, \theta, k)=\max _{\sigma \in[0, \bar{\sigma}(\alpha, \beta)]} k\left[\mu \theta P\left(s_{b \vee r}=1\right)+\theta P\left(s_{b \vee r}=0.5\right)-\sigma\right], \tag{OA.19}
\end{equation*}
$$

where $P\left(s_{b \vee r}=1\right)=\alpha e_{b}+\alpha e_{r}-\alpha^{2} e_{b} e_{r}$ and $P\left(s_{b \vee r}=0.5\right)=e_{b}+e_{r}-e_{b} e_{r}(1+2 \alpha)$, subject to

$$
\begin{align*}
& e_{b}(\sigma, \alpha, \beta)=\sigma(1+\alpha) \frac{0.5-\beta+2 \beta \sigma(0.5+\beta)\left(1+\alpha+\alpha^{2}\right)}{1+4 \sigma^{2} \beta^{2}\left(1+\alpha+\alpha^{2}\right)^{2}}  \tag{OA.20}\\
& e_{r}(\sigma, \alpha, \beta)=\sigma(1+\alpha) \frac{0.5+\beta-2 \beta \sigma(0.5-\beta)\left(1+\alpha+\alpha^{2}\right)}{1+4 \sigma^{2} \beta^{2}\left(1+\alpha+\alpha^{2}\right)^{2}} \tag{OA.21}
\end{align*}
$$

Figure OA. 8 shows the optimal principal's profit as a function of the productivity-cost ratio $\theta$, for different levels of subtle bias ( $\beta_{1}=0$ and $\beta_{2}=0.5$ ) and different levels of the parameters $\alpha$ and $\mu$. As we have shown in the main text, for low productivity (low- $\theta$ firms), the principal is better off under high levels of subtle discrimination $(\beta=0.5)$, while for high levels of productivity (high- $\theta$ firms), she is better off under no discrimination $(\beta=0$ ). Note that the region where high subtle discrimination is optimal from the firm's point of view is decreasing in $\alpha$. In other words, subtle discrimination becomes less profitable for firms as their workers obtain high skill with a greater chance, conditional on an effort level. Figure OA. 9 illustrates this insight. It depicts the threshold productivity-cost ratio $\theta^{\prime}$ such that for $\theta<\theta^{\prime}$, the principal's profit is maximized under the highest level of subtle discrimination, $\beta^{p m}=0.5$ and for $\theta>\theta^{\prime}$ it is maximized under no subtle discrimination, $\beta^{p m}=0$ as a function of parameter $\alpha$ and for two different values of parameter $\mu$. Note that $\theta^{\prime}$ decreases in both $\alpha$ and $\mu$.


Figure OA.8: Equilibrium profit as a function of the premium-cost ratio, $\sigma$, for different values of the subtle bias $\beta$ and the parameters $\alpha$ and $\mu$.

## OA.8.3 Principal's choice of skill assessment precision

In this subsection, we investigate under what conditions the principal prefers a more granular skill partition. For example, if an agent is highly skilled, $s_{i}=1$, the principal might need to pay an extra cost to distinguish such agent from a skilled agent $s_{i}=0.5$.

We assume that promoting a moderately skilled agent increases the principal's profit by $H$, while promoting a highly skilled agent increases his profit by $\mu H$, where $\mu>1$. Below


Figure OA.9: Threshold $\theta^{\prime}$ as a function of $\alpha$ for different values of the parameter $\mu$.
we compare the principal's profit when agents' skill can take three levels: $s_{i}=\{0,0.5,1\}$, when the principal distinguishes between all the three levels and when he distinguishes only skilled ( $s_{i}=0.5$ and $s_{i}=1$ ) and non-skilled agents ( $s_{i}=0$ ).

We solve the principal's problem under no subtle discrimination. First, the principal's profit when he recognizes three levels of skill is given by Eq. (OA.19-OA.21), where $\beta=0$. Second, let us consider a scenario wherein the principal is aware of the existence of highly productive agents with $s_{i}=1$, but he lacks the ability (or intentionally chooses not) to differentiate between individuals possessing high skill levels ( $s_{i}=1$ ) and those with medium skill levels $\left(s_{i}=0.5\right)$. In this case, agent $i$ faces a problem similar to the one in Eq. (4) of the main text:

$$
\begin{equation*}
\max _{e_{i}} \sigma\left[(1+\alpha) e_{i}\left(1-(1+\alpha) e_{-i}\right)+\frac{1}{2}\left(1-(1+\alpha) e_{i}-(1+\alpha) e_{-i}+2(1+\alpha)^{2} e_{i} e_{-i}\right)\right]-\frac{e_{i}^{2}}{2}, \tag{OA.22}
\end{equation*}
$$

because when an agent chooses effort level $e_{i}$, she becomes skilled (either moderately skilled or highly skilled) with probability $e_{i}+\alpha e_{i}$. With no subtle discrimination, the reaction functions are the same as when the principal can distinguish between all three skill levels $: e_{b}^{*}=e_{r}^{*}=\frac{\sigma}{2}(1+\alpha)$.

The principal's problem in this case is

$$
\begin{equation*}
\pi(k, \theta, \mu, \alpha)=\max _{\sigma \in[0, \sigma(\alpha)]} k\left[\theta\left(e_{b}+e_{r}-e_{b} e_{r}\right)\left(\mu \frac{\alpha}{1+\alpha}+\frac{1}{1+\alpha}\right)-\sigma\right] \tag{OA.23}
\end{equation*}
$$

subject to $e_{i}=\frac{\sigma}{2}(1+\alpha)$.
Figure OA. 10 shows the principal's optimal profit for cases when she can distinguish
between three skill levels (red lines) and for cases when she can only distinguish between two skill levels (black lines). We plot the principal's profit for several values of $\alpha$ and $\mu$.

(a) $\alpha=0.2$

(b) $\alpha=0.4$

Figure OA.10: Equilibrium profit as a function of the productivity-cost ratio, $\theta$, for different values of $\mu$ and $\alpha$. Red lines correspond to cases when the principal distinguishes between three skill levels, while black lines correspond to cases when the principal cannot distinguish between medium and highly skilled agents at the promotion stage.

The results in Figure OA. 10 show that the principal's profit is higher when he possesses an ability to distinguish between all three skill levels. As a result, the principal would be willing to pay a fixed cost in order to obtain this ability. Importantly, the principal's willingness to pay is increasing in the productivity-cost ratio $\theta$. That is, in low-productivity firms, the principal may decide not to separate between moderately and highly skilled agents if such separation is costly. For example, one might need to conduct additional rounds of interviews to identify highly skilled candidates. If promotion of highly skilled candidates is not profitable enough, the firm may choose to conduct fewer rounds of interviews and to "pool" moderately and highly skilled agents in promotions.

## OA. 9 Welfare analysis

## OA.9.1 Summary

Subtle discrimination simplifies welfare and policy analyses because welfare comparisons are not confounded by the direct effects of biases on the principal's utility. This property allows our model to produce sharper welfare and policy implications. Here we show that, for moderate to high stakes, subtle discrimination may harm everyone: the favored agent,
the unfavored agent, and the firm. However, and perhaps surprisingly, for sufficiently low stakes, everyone may benefit from subtle discrimination. This result arises because lowproductivity firms use biased contests as a means to incentivize agents.

In this section, we address a number of normative questions: How does the equilibrium compare iwth the first-best? What are the welfare implications of subtle discrimination? When is subtle discrimination inefficient?

## OA.9.1. 1 Comparison with the first best

It is instructive to compare the equilibrium effort levels for a given $W$ to their first-best counterparts. For $\beta>0$, there is typically no contract that implements the first-best investment levels. If $H>k$, the first-best outcome is $e_{b}^{F B}=1$ and $e_{r}^{F B}=0$. This outcome is unachievable under subtle discrimination: From Proposition 1, to have $e_{b}^{*}=1$ we need $\sigma \geq \bar{\sigma}$, in which case we have $e_{r}^{*}=\min \left\{\frac{\sigma(1-2 \beta)}{2}, 1\right\}>0$ (because $\beta<0.5$ if $\bar{\sigma}$ is finite). If $H \leq k$, the first-best requires both agents to invest $\tilde{e}=\frac{H}{H+k}$. But agents' investments are the same if and only if $\sigma=1$, in which case we have $e_{r}^{*}=e_{b}^{*}=0.5 \geq \tilde{e}$. Thus, except for the case in which $H=k$, there is no $\sigma$ that implements the first-best investment levels in the presence of subtle bias $(\beta>0)$.

Things are different under no subtle bias $(\beta=0)$. If $H \leq k$, the first-best can be achieved by choosing $\sigma^{F B}=\frac{2 H}{H+k}$ (i.e., $W^{F B}=\frac{2 k H}{H+k}$ ). If $H>k$, the first-best cannot be achieved.

To summarize: (i) if the principal is subtly biased, there is no contract that implements the first-best outcome, except for the (measure-zero) case in which $H=k$; (ii) if the principal is unbiased, the first-best outcome can be implemented by a suitably-designed promotion contest if and only if $H \leq k$. The comparison with the first-best shows that subtle discrimination is a friction. Without a subtle bias, the first-best can sometimes be achieved. If there are additional contractual frictions, subtle discrimination can nevertheless be welfare-enhancing in some cases, as we show next.

## OA.9.2 When does discrimination harm workers?

Figures OA.11a and OA.11b show the utilities of blue and red agents as functions of the productivity-cost ratio, $\theta$, and the subtle bias, $\beta$, when the contract $\sigma(\theta, \beta)$ is optimally chosen by the principal. Two features are worth highlighting.

First, a stronger bias is not always beneficial to Blue. For high $\theta$, increasing the bias may decrease Blue's utility. How could a bias in favor of blue agents harm these exact
agents? A more biased principal offers lower stakes, reducing the benefits of promotion. As the figure shows, this dampening of incentives can offset Blue's gains from a higher bias. Thus, since profits may decrease with the subtle bias, there exist regions in which reducing the bias is a strict Pareto improvement, even in the absence of side transfers.

Second, there exists a region (for small values of $\theta$ ) where the red agent prefers more discrimination to less. Therefore, for low levels of the productivity-cost ratio, all (the principal and both agents) prefer more discrimination to less. This result highlights that players at different layers of the corporate hierarchy, as well as in different industries, are heterogeneous in their preferences with respect to anti-discriminatory policies. While in positions or industries where productivity gains upon promotion are high everyone may benefit from decreased discrimination, this is not always the case in positions or industries with low productivity gains.


Figure OA.11: Agents' utilities, $U^{*}$, under optimal contract $\sigma(\theta, \beta)$.

## OA.9.3 Social surplus

Figure OA. 12 presents the level of subtle bias that maximizes the total social surplus, $S$, as a function of the productivity-cost ratio, $\theta$. The relationship between subtle bias and social surplus is complex. There are three regions. In the first region, low- $\theta$ firms benefit from high subtle biases because the overcompensation effect helps to incentivize red agents. As we see from Figure OA.11b, for sufficiently low values of $\theta$ both Blue and Red benefit from increasing the bias. ${ }^{5}$ In the second region, Red no longer benefits from the bias and,

[^16]eventually, the discouragement effect becomes dominant, thus the firm also prefers a lower bias. Thus, for firms with intermediate levels of $\theta$, the social-surplus-maximizing bias is $\beta=0$. In the third region, Blue's utility is hump-shaped in the subtle bias (see Figure OA.11b), while the firm's profit is relatively flat in $\beta$. The optimal bias trades off the gains and losses to the agents. The socially-optimal bias is increasing in the productivity-cost ratio because discouraging Red is efficient when Blue is more likely to win, as it reduces the deadweight costs of effort duplication.


Figure OA.12: Socially optimal level of subtle bias as a function of $\theta$.

## OA. 10 Quotas

## OA.10.1 Summary

We use our model to investigate the consequences of a hard quota aimed at protecting the unfavored group. We show that the quota has its desired effect only when the bias is sufficiently high. Even in that case, despite the fact that the quota implements equality of outcomes, unfavored agents still fare worse in terms of expected utility than favored agents. This result is explained by firms reducing their promotion stakes under the quota, which leads to unfavored agents working harder than favored agents.

We have different results for soft (i.e., voluntary and non-binding) quotas. While potentially quite effective at curbing subtle discrimination, our results show that only highproductivity firms choose to implement soft quotas.

## OA.10.2 Introducing a hard quota

The analysis in Section 3.5 of the main text reveals that not all firms would voluntarily take steps towards reducing subtle biases. At the same time, the welfare analysis shows that reducing subtle biases is sometimes socially desirable. Thus, it is instructive to consider possible interventions aimed at reducing or eliminating subtle discrimination.

Setting a (hard) quota is a popular policy tool to tackle a lack of diversity at top positions. Quotas are unlikely to deliver efficiency gains in our model, for two reasons: they constrain the principal's maximization problem and directly interfere with the agents' incentives to invest. Nevertheless, quotas may be a policy option for reasons other than efficiency, such as equity and fairness.

To consider quotas at the firm level, we extend the model as follows. At Date 0, the firm has a continuum of vacancies for job 1 , with mass $2 \mu$, and for job 2 , with mass $\mu$. Each worker in job 1 competes with exactly one worker for promotion and all pairs of workers are mixed (one red and one blue). In equilibrium, the probability that an agent of type $i$ is promoted, $p_{i}$, is also the proportion of agents of type $i$ found in job 2 at the end of the game. A quota is a target for $p_{i}$ or, equivalently, a target for the promotion gap, $\Delta p$. For convenience we use the latter, thus a quota is fully described by a number $q \in[-1,1]$.

Without loss of generality, we assume that the quota's goal is to reduce the promotion gap, that is, to promote more red agents: $q<\Delta p_{0}$ (the pre-quota promotion gap). Our interpretation is that the principal designs a firm-wide promotion policy, which is then implemented by a mass $\mu$ of supervisors, one for each pair of workers in job 1 . We assume that supervisors have incentives aligned with the firm but are subtly biased. Here, unlike in Subsection 3.5, the firm cannot choose the bias of its supervisors. Because only they observe the skill $s_{i}$ of their pairs of subordinates, any rule that allows supervisors discretion can be abused. Thus, the only way to comply with the quota is to force some supervisors to promote red agents regardless of skill. To do so, the principal offers a proportion $\delta$ of supervisors discretion over promotion decisions and forces a proportion $1-\delta$ of supervisors to promote only red agents.

The principal chooses $\delta$ to maximize profit subject to the quota constraint, $\Delta p=q$. The principal has two options: he can reveal the identities of the "constrained" and "unconstrained" supervisors to their subordinates, or he can keep them secret. For brevity, we
only consider the full disclosure case. ${ }^{6}$ The principal's problem is

$$
\begin{equation*}
\Pi(\beta, \theta, q)=\max _{\sigma \in[0, \bar{\sigma}(\beta)], \delta \in[0,1]} \delta \theta\left(e_{b}+e_{r}-e_{b} e_{r}\right)-\sigma \tag{OA.24}
\end{equation*}
$$

subject to

$$
\begin{gather*}
e_{b}=\frac{\sigma(0.5-\beta)+2 \beta \sigma^{2}(0.5+\beta)}{1+4 \beta^{2} \sigma^{2}} ; e_{r}=\frac{\sigma(0.5+\beta)-2 \beta \sigma^{2}(0.5-\beta)}{1+4 \beta^{2} \sigma^{2}} ;  \tag{OA.25}\\
\Delta p \equiv \delta\left\{e_{b}-e_{r}+\left[e_{b} e_{r}+\left(1-e_{b}\right)\left(1-e_{r}\right)\right] 2 \beta\right\}-(1-\delta)=q \tag{OA.26}
\end{gather*}
$$

where the last equation is the quota constraint: the promotion gap must be $q$.
Firm profit is always higher when there is no quota or if the quota is not binding (i.e., $q=\Delta p_{0}$ ) because the quota constrains the principal's maximization problem. Still, there might be reasons to support quotas on grounds of redistributive equity. The key question is then: when do discriminated agents benefit from quotas?

(a) For $\theta_{1}=2.0, \beta=0$ is socially optimal and $\beta=0.5$ is profit maximizing

(b) For $\theta_{2}=3.2, \beta=0$ is both socially optimal and profit maximizing

Figure OA.13: Agents' utilities as functions of subtle bias under no quota and under a fully disclosed quota, $\Delta p=q=0$, for $\theta_{1}=2.0$ and $\theta_{2}=3.2$.

Figure OA. 13 shows the utilities of Blue and Red under a $50 \%$ quota (i.e., $q=0$ ). As expected, the quota typically reduces Blue's utility and increases Red's utility. However, for low biases, the quota may reduce Red's utility. This counter-intuitive result occurs because, under a quota, the firm offers smaller stakes. The negative effect dominates when the bias is small because, in this case, Red's probability of promotion increases by only a small amount after the quota.

[^17]We also see that the favored agent is typically better off than the discriminated agent, even when the quota imposes full parity. For Red to do better than Blue under a quota, the bias must be small and productivity $\theta$ must be high.

## OA.10.3 Soft quotas

Firms may be able to achieve their diversity goals through voluntary actions, such as the adoption of a soft quota (or "soft affirmative action," as in Fershtman and Pavan (2021)). Rather than setting a strict numeric target, we can think of soft quotas as a recommendation to promote more red agents whenever possible. Suppose that, to implement a soft quota, the firm adopts a policy in which a supervisor pays a (vanishingly) small cost $\kappa$ every time they promote a blue agent. For example, the supervisor needs to write a report explaining why the blue agent was more qualified than the red agent. As long as $\kappa$ is sufficiently small and supervisors have strong incentives to maximize firm profit, the soft quota would only affect supervisors' behavior in tie-breaking situations. What types of firms adopt soft quotas? The answer follows from Proposition 7:

Corollary 1. The firm adopts a soft quota that incentivizes the promotion of red agents if and only if $\theta \geq \theta^{\prime}$.

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[^1]:    ${ }^{1}$ See, for example, Dovidio and Gaertner (1986), Essed (1991), Deitch et al. (2003), Dipboye and Halverson (2004), Noh et al. (2007), Van Laer and Janssens (2011), Jones et al. (2017), Dhanani et al. (2018), and Hebl et al. (2020). While studies often use alternative terms, such as "modern discrimination," "aversive discrimination," "everyday discrimination," "ambivalent discrimination" and "covert discrimination," they all contrast subtle discrimination with "old-time" overt discrimination and emphasize its ambiguous, hard-to-detect and yet pernicious nature.

[^2]:    ${ }^{2}$ Despite women's better performance, supervisors still consider men to have higher "potential" on average, which leads to higher promotion rates for men.

[^3]:    ${ }^{3}$ Bircan et al. (2022) show evidence of correct belief formation about women's disadvantage in the assignment of roles.

[^4]:    ${ }^{4}$ This is similar to Athey et al. (2000), who also assume a diverse entry-level workforce in a model of promotions. In Section OA.5, we present an extension with an explicit labor market model.
    ${ }^{5}$ The model requires only that the principal observes $s_{i}$, thus the "non-verifiability" of $s_{i}$ is not a crucial assumption. However, assuming that all parties observe $s_{i}$ makes the interpretation more natural. See also Section OA. 3 for a variation of the model with continuous skills and another version of the model (Section OA.8) with endogenous skill assessment precision (as in Meyer (1991)).
    ${ }^{6}$ Our model differs from mentoring models in which workers of the disadvantaged type may find it more costly to acquire firm-specific skills than do workers of the advantaged type; see Athey et al. (2000), Müller-Itten and Öry (2022), and Cabral (2022).

[^5]:    ${ }^{7}$ Although we interpret this decision as a promotion, it can also be interpreted as a direct hiring decision for a specific post. See Coate and Loury (1993) for a model that allows for both interpretations. For a related model of worker competition for job slots, see Lazear et al. (2018).
    ${ }^{8}$ For a theory and recent evidence on the importance of human capital acquisition for career progression inside firms, see Pastorino (2022).

[^6]:    ${ }^{9}$ Our decision-making heuristic can be mapped into Tversky's (1969) notion of lexicographic semiorder; see also Manzini and Mariotti (2012) for a generalization. In such models, ties can arise with positive probability even when the assessment criteria are continuous.

[^7]:    ${ }^{10}$ In Subsection 3.4, we show that optimal contracts always imply interior solutions.

[^8]:    ${ }^{11}$ The flatness of the reaction functions under $\beta=0$ is a robust feature and not a consequence of the binary setup (see Section OA. 8 for an example with more than two skill levels). The key assumption here is that low-skill agents are sometimes promoted.
    ${ }^{12}$ Although we do not rely on any notion of stability, we note that the unique equilibrium is globally stable under

[^9]:    continuous adjustments. Under discrete adjustments, the equilibrium is globally stable if and only if $\beta \sigma<0.5$.
    ${ }^{13}$ See Coate and Loury (1993) and MacLeod (2003) for early models with a similar discouragement effect.

[^10]:    ${ }^{14}$ To do so, the principal could construct a narrative that is not contradicted by his observed data (see, for example, Eliaz and Spiegler (2020)) rather than engage in rational Bayesian learning.

[^11]:    ${ }^{15}$ Glover et al. (2017) provide evidence of a different form of self-fulfilling discrimination: workers perform worse when under the supervision of a biased manager.

[^12]:    ${ }^{16}$ Kline et al. (2022) show that only a small fraction of U.S. employers discriminate. However, these models suggest that even a small bias can have substantial consequences for the economy.

[^13]:    ${ }^{1}$ Alternatively, Bohren et al. (2019) show how to test for the source of discrimination by analyzing the implications of a dynamic model of discrimination.
    ${ }^{2}$ For example, Benson et al. (2021) and Huang et al. (2022) carry out the Becker outcome test in promotion contexts.

[^14]:    ${ }^{3}$ This is obviously not necessary for the results, but it helps make the point that there is nothing special about the "ties" in our main model.

[^15]:    ${ }^{4}$ For a model of poaching along these lines, see Ferreira and Nikolowa (2023).

[^16]:    ${ }^{5}$ Note that the bias itself does not directly affect utilities. Thus, our welfare results fundamentally differ from those of models with non-subtle biases. For example, in Prendergast and Topel (1996), an increase in bias directly benefits supervisors.

[^17]:    ${ }^{6}$ The no disclosure case yields similar results.

